MAT 250B HW01

[add your name here]

Due Friday, 1/12/24 at 11:59 pm on Gradescope

Reminder. *Your homework submission* **must be typed** *(TeX'ed) up in full sentences, with proper mathematical formatting.*

The following resources may be useful as you learn to use TeX and Overleaf:

- *Overleaf's introduction to LaTeX:*
  https: // www. overleaf. com/ learn/ latex/ Learn_ LaTeX_ in_ 30_ minutes

- *Detexify:*
  https: // detexify. kirelabs. org/ classify. html

The goal of this first "homework" is to make sure everyone knows how to typeset and submit homework. You can write as much or as little as you'd like for each answer; just make sure to submit a solution for every problem.

Exercise 1

What's your name? Any nicknames? What are your pronouns? What is your intended area of study? Anything else you want to tell me?

SOLUTION.

Exercise 2

I'm assuming you remember basic undergraduate algebra, including terms like

> group, subgroups, cosets, conjugacy, group actions, normal, rings, ideals, fields, product, quotient, homo/iso/auto/endo-morphisms of groups/rings/fields, abelian, classification of finitely generated abelian groups, dihedral, cyclic, symmetric/permutation groups, order, index, orbit, stabilizer, center/central, integral domains, fraction fields, PIDs

Here are some topics you covered in Prof. Kuperberg's 250A course:

> group presentations, kernel/cokernel, Smith normal form, torsion, free groups, torsion, p-group, Sylow theorems and how to use them, Jordan-Holder, composition series, solvable, perfect, lower/upper central series, nilpotent, basic category theory, rings, modules, Noetherian/Artinian rings, PIDs, algebras (over a ring), prime ideals, Euclidean domain, UFD, classification of finitely generated modules over a PID, Jordan canonical form

The first step to being good at algebra is **knowing the precise definitions of the terms that pop up**. This step is non-negotiable. So, take a look at the terms above, and figure out which look totally foreign or are terms/concepts/theorems where, if I asked you what it is, you wouldn't be able to tell me. I'm not saying to spend a super long time reviewing all the above – we just need to start by remembering the definitions. The terms above are also not necessarily a comprehensive foundation for the algebra we will be studying this quarter. However, you will at some point in grad school be expected to know what all of these terms mean!

The second thing I need you to do is to **let me know when I'm using a term or concept you are not familiar with, especially if you are a graduate student**. If enough students are not familiar with a topic that I'm assuming you know, then that's something we need to cover in some form in this class. Also, by letting me know you're not comfortable with a topic, I can let you know how important it is that you go learn it, at least for the purposes of this quarter.

(a) Of the topics listed above, are there any that you feel you need to review again?

(b) Are there any major topics you feel I have left out of the lists above, that you feel have been covered in detail in 250A?

SOLUTION.

Exercise 3

Please discuss how familiar you are with the following topics. For example, did you take an undergrad-level course on this in the past? Would you be able to take a prelim-level test on the topic right now? What about after 1 hour of review?

Note: You are *by no means* expected to know these already. Part of being a good mathematician is being able to accurately gauge your own knowledge of a topic. This can be hard before you know much about the topic, but I want you to try introspecting on this. (I'm also asking this question for my own lesson planning purposes, of course.)

(a) Galois theory

(b) Representation theory of finite groups

(c) Lie groups, Lie algebras

Exercise 4

This is a graduate-level course in mathematics, which means that you will need to spend a lot of time outside of class thinking about the material we cover in class. In fact, these may be the last few years of your life when you learn in a classroom setting, and it's important that you also learn (1) how you learn, (2) how to learn from a textbook in conjunction with talking to people, and (3) how to navigate and synthesize multiple resources.

However, I know that the time you have is not infinite. You will need to figure out what to prioritize, and how to learn most efficiently. I'm sure you know this, but just in case: you should start the homework early, try everything, get stuck, and ask for help. You are expected to do the homework, and you also not expected to immediately know how to do all the homework just from attending lecture.

With all that said, consistency is the key to success. Take a moment to consider how many hours you **realistically** will have to focus on this course (outside of lectures) each week. (This should help you prioritize tasks related to your learning.)

Roughly how many hours do you think you will have to focus on this course? What types of activities do you think will help you learn the material most efficiently? (E.g. pre-skimming the text, doing the homework, study groups, attending office hours, etc.)

SOLUTION.

Exercise 5

Finally, let's try typesetting some stuff, just to make sure you'll be able to type up upcoming homeworks.

Prove that if $\varphi : R \to S$ is a ring homomorphism, then the kernel $\ker \varphi = \varphi^{-1}(0_S)$ **is a 2-sided ideal of** R**.**

SOLUTION.