MAT 250B Final Exam Information

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1 Topics Covered

Here's a list of topics I remember covering in this class:

- Chapter 6 of Rotman: modules; products and coproducts, pushout and pullback; free, projective, injective, flat modules; bilinear maps and tensor products
  - We also covered limits and colimits, but this will not be covered in the final, and was just for your mathematical culture.
- §8.5, 8.6 of Rotman: bilinear forms; tensor and exterior algebras, multilinear maps; quadratic forms
  - We didn't cover symmetric algebras but you are prepared to read about them now.
- §2.9, 3.1, 3.2; §5.4.3 of Rotman / Chapters 13 and 14 of Dummit and Foote: field extensions (mainly algebraic), algebraic closures, finite fields, Galois theory
  - We didn't cover transcendence degree. You should look at the definition for your mathematical culture.

2 Practice Problems

Here are some problems that were rejected while writing the exam. I don't plan on posting solutions, but can provide hints at office hours or on the class website if needed.

1. Prove that the $\mathbb{Z}$-module $\mathbb{Q}/\mathbb{Z}$ is injective but not flat.
2. Let V be a real vector space. Prove that the set of bilinear forms $V \times V \to \mathbb{R}$ has the structure of a real vector space.
3. Prove that an algebraically closed field must be infinite.
4. Factor $x^8 - x$ into irreducibles in $\mathbb{Z}[x]$ and $\mathbb{F}_2[x]$.
5. Determine the Galois group of $x^3 - 2x + 4$ over $\mathbb{Q}$ and $\mathbb{F}_3$.