MAT 150A HW03

[add your name here]

Due Tuesday, 1/30/24 at 11:59 pm on Gradescope

Instructions Solve the following problems, and then type up your solutions in full sentences after the

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\solution
```

command following each exercise. It may help to look at how I typed the exercise, e.g. to learn the command used to typeset a particular symbol. Compile often. See the instructions in HW00 if you're unsure how to use Overleaf.

Proof-based course This is a proof-based course and you are expected to **clearly prove** all your claims. If you're wondering how much detail to include, a good rule of thumb is that your proofs should be slightly more detailed than the proofs in the book, but not less detailed. They should also not be unreasonably verbose.

Reminder Homeworks must be typed using LaTeX **in full sentences with proper mathematical formatting**. Handwritten homeworks will not be accepted. If there is a documented reason why you can't type up your homework, let me know and we can discuss an alternate policy. Otherwise, please consider learning how to properly write and typeset mathematics as part of this course.

The following resources may be useful as you learn to use TeX and Overleaf:

- Overleaf's introduction to LaTeX: `https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes`
- Detexify: `https://detexify.kirelabs.org/classify.html`

Grading Most (parts of) problems will be graded for completion out of 5 points. A few selected problems will be graded more carefully; these will be revealed after the homework is due. Abridged solutions will be posted after the 24-hour grace period after the homework due date.

Exercise 1

Prove that every subgroup of a cyclic group is cyclic. *Hint: Work with exponents and use the description of the subgroups of* $\mathbb{Z}^+$.

Solution.

Exercise 2

Let a, b be elements in a group G. We say a is **conjugate** to b if there exists $g \in G$ such that $b = gag^{-1}$. Prove that **conjugacy** is an equivalence relation.

Solution.

Exercise 3

Prove that equivalence relation $\sim$ on a set S determines a partition P, and vice versa.

Solution.

Exercise 4

Why is following assignment **not** a well-defined function between sets?

$$\begin{aligned} \varphi : \mathbb{Z}/10\mathbb{Z} &\to \mathbb{Z}/7\mathbb{Z} \\ \bar{k} &\mapsto \bar{k} \end{aligned}$$

SOLUTION.

Exercise 5

Let $\varphi : G \to G'$ be a homomorphism.

(a) Prove that $\ker \varphi$ is a subgroup of G.

(b) Prove that $\operatorname{im} \varphi$ is a subgroup of G'.

(c) Prove that $\ker \varphi = \{1_G\}$ if and only if φ is injective (as a set map).

SOLUTION.