MAT 150A HW05

[add your name here]

Due Tuesday, 2/13/24 at 11:59 pm on Gradescope

Proof-based course This is a proof-based course and you are expected to **clearly prove** all your claims. If you're wondering how much detail to include, a good rule of thumb is that your proofs should be slightly more detailed than the proofs in the book, but not less detailed. They should also not be unreasonably verbose.

Reminder Homeworks must be typed using LaTeX **in full sentences with proper mathematical formatting**. Handwritten homeworks will not be accepted. If there is a documented reason why you can't type up your homework, let me know and we can discuss an alternate policy. Otherwise, please consider learning how to properly write and typeset mathematics as part of this course.

Exercise 1

For $a \in G$, define the map *conjugation by a* by

$$\begin{aligned} c_a : G &\to G \\ x &\mapsto axa^{-1}. \end{aligned}$$

Prove that c_a is an *automorphism* of G.

SOLUTION.

Exercise 2

Let $\varphi : G \to G'$ be an isomorphism of groups. Since φ is a bijection, there exists an inverse *set* map φ^{-1}. Prove that φ^{-1} is also a (group) isomorphism.

SOLUTION.

Exercise 3

Let q be a 5-cycle in S_9.

(a) What is the cycle type of q^{17}?

(b) How big is the conjugacy class of q?

SOLUTION.

Exercise 4

Let G be a group, and let $a, b \in G$. Prove that ab and ba are conjugate elements.

Solution.

Exercise 5

(Kernels are normal) Let $\varphi : G \to G'$ be a homomorphism. Prove that $\ker \varphi \trianglelefteq G$.

Solution.

Exercise 6

Let K and H be subgroups of a group G.

(a) Prove that the intersection $K \cap H$ is a subgroup of G.

(b) Prove that if $K \trianglelefteq G$, then $K \cap H \trianglelefteq H$.

(c) Prove that if both $K, H \trianglelefteq G$, then $K \cap H \trianglelefteq G$.

Solution.