MAT 150A HW07

[add your name here]

Due Tuesday, 2/27/24 at 11:59 pm on Gradescope

Proof-based course This is a proof-based course and you are expected to **clearly prove** all your claims. If you're wondering how much detail to include, a good rule of thumb is that your proofs should be slightly more detailed than the proofs in the book, but not less detailed. They should also not be unreasonably verbose.

Reminder Homeworks must be typed using LaTeX **in full sentences with proper mathematical formatting**. Handwritten homeworks will not be accepted. If there is a documented reason why you can't type up your homework, let me know and we can discuss an alternate policy. Otherwise, please consider learning how to properly write and typeset mathematics as part of this course.

Exercise 1

Leet G and G' be cyclic groups of orders 12 and 6, generated by elements x and y, respectively. Let $\varphi : G \to G'$ be the map defined by $\varphi(x^i) = y^i$. Exhibit the correspondence referred to in the Correspondence Theorem **explicitly**.

Solution.

Exercise 2

Suppose $\varphi : G \to G'$ is a surjective homomorphism, and that subgroups $H \leq G$ and $H' \leq G'$ correspond to each other under the bijection in the Correspondence Theorem. Prove that $[G : H] = [G' : H']$.

Solution.

Exercise 3

Prove that a conjugate of a glide reflection in $\text{Isom}(\mathbb{R}^2)$ is also a glide reflection, and that the glide vectors have the same length.

Solution.

Exercise 4

We used $\mathbb{R}^2$ to describe the points on the plane. We could equivalently use $\mathbb{C}$, the complex plane. Since we use the same notion of distance for points in the complex plane, as metric spaces, $\mathbb{R}^2$ is the same as $\mathbb{C}$. The generators of $\text{Isom}(\mathbb{R}^2) = \text{Isom}(\mathbb{C})$ on page 160, Equation (6.3.1), in the textbook.

Write formulas for the generators of $\text{Isom}(\mathbb{C})$ in terms of the complex variable $z = x + iy = re^{i\theta}$.

Solution.