MAT 150A HW09

[add your name here]

Due Tuesday, 3/12/24 at 11:59 pm on Gradescope

Proof-based course This is a proof-based course and you are expected to **clearly prove** all your claims. If you're wondering how much detail to include, a good rule of thumb is that your proofs should be slightly more detailed than the proofs in the book, but not less detailed. They should also not be unreasonably verbose.

Reminder Homeworks must be typed using LaTeX **in full sentences with proper mathematical formatting**. Handwritten homeworks will not be accepted. If there is a documented reason why you can't type up your homework, let me know and we can discuss an alternate policy. Otherwise, please consider learning how to properly write and typeset mathematics as part of this course.

Exercise 1

Does the rule $P * A = PAP^\top$ define an operation of GL_n on $M_{n\times n}$, the set of $n \times n$ matrices? *Here, $P^\top$ is the transpose of the matrix $P \in GL_n$.*

Exercise 2

Suppose a group G **acts freely** on a set S (i.e. the group action $G \curvearrowright S$ is free). Prove that for any $s \in S$, the stabilizer G_s is the trivial subgroup of G.

Exercise 3

What is the stabilizer of the coset $[aH]$ for the action of G on G/H?

Exercise 4

Let $G = GL_n(\mathbb{R})$ act on the set $V = \mathbb{R}^n$ by left multiplication.

(a) Describe the decomposition of V into orbits for this action.

(b) What is the stabilizer of e_1?

(c) Is this action of G on $V - \{0\}$ free, transitive, both, or neither?

Exercise 5

Let G be the group of rotational symmetries of a cube. Let V, E, and F denote the set of vertices, edges, and faces of a cube, respectively. Check for yourself that the size of these sets are

$$|V| = 8 \quad |E| = 12 \quad |F| = 6.$$

Fix a vertex $v \in V$, an edge $e \in E$, and a face $f \in F$, and let G_v, G_e, and G_f be their stabilizers, respectively.

Determine the formulas of the form

$$|S| = |O_1| + |O_2| + \cdots + |O_k|$$

(formula 6.9.4 in the text) that represent the decomposition of each of the three sets V, E, F into orbits under the action of each of the subgroups of G.

Note Your solution should contain $9 = 3 \times 3$ formulas, one for each (group, set) pair, such as $G_v \curvearrowright V$, $G_v \curvearrowright E$, etc. You should explain any geometric reasoning in words; if you'd like, you can also include a picture by following the instructions here. The tl;dr is that you should use the following code:

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\includegraphics[width=4cm]{yourImageName}
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