MAT 150A Winter 2024
Instructor: Melissa Zhang
Exam 1

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): ____________________ Name (print): ____________________

Name of left neighbor: ____________ Name of right neighbor: ____________

If you are next to the wall, then write "Wally" as your left or right neighbor. Write "Nemo" for your left/right neighbor if you don't have a left/right neighbor, respectively.

Question	Points	Style Points	Score
Q1	20	5	
Q2	15	5	
Q3	15	5	
Q4	30	5	
Total:	80	20	

- This is a **closed-book** exam. You may not use the textbook, cheat sheets, notes, or any other outside material. No calculators, computers, phones, or any other electronics are allowed.
- The last page of the exam packet is provided for scratchwork. **Do not detach** this sheet from your exam packet.
- You have **45 minutes** to complete this exam. If you are done early, you may leave after handing in your exam packet.
- Everyone must work on their own exam. Any suspicions of collaboration, copying, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.
- **This is a proof-based course.** All statements must be justified and argued **clearly** in the style of a mathematical proof. Failure to do so will result in the loss of correctness and/or style points.

1. Consider $H = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \,\middle|\, x \in \mathbb{R} \right\} \subset GL_2(\mathbb{R})$. Prove that H is a subgroup of $GL_2(\mathbb{R})$. *Recall that $GL_2(\mathbb{R})$ is a group under matrix multiplication.*

2. Let G be a group, and let x and g be elements of G.

   (a) First suppose that $|x| = n < \infty$. Prove that $|x| = |gxg^{-1}|$.
   (b) Now suppose $|x| = \infty$. Prove that we also have $|x| = |gxg^{-1}|$.

3. Let G be a group. Prove that $\varphi : G \to G$ defined by $\varphi(x) = x^{-1}$ is a homomorphism if and only if G is abelian.

4. Let $f : \mathbb{C}^\times \to \mathbb{C}^\times$ be defined by $z \mapsto z^4$. *Recall that the group operation on* $\mathbb{C}^\times = \mathbb{C}^\times - \{0\}$ *is complex multiplication.*

   (a) Prove that f is a homomorphism.

   (b) Find the kernel of f.

   (c) Find the image of f.

Scratchwork

Nothing on this page will be graded.