MAT 150A Exam 2 Information

Instructor: Melissa Zhang

Exam date/time: Wednesday, November 13, 2023; in class

Topics Covered

Exam 1 will cover the material corresponding to

- Lectures 1–19

- Mainly textbook sections 2.5–2.11, 7.9, 7.10

- Homeworks HW01 through HW05

Some important terms and concepts we covered include but are not limited to:

- product and quotient groups

- conjugation (in S_n and in general)

- group presentations, generators and relations

- Correspondence Theorem

- First Isomorphism Theorem

How do I study for the exam?

Reading over the slides or notes can give you a false sense of understanding. The best way to test your knowledge of the concepts is practice doing exercises.

1. Go over the homework solutions. What concepts did you struggle on? Did you have a misunderstanding of any of the covered topics? Focus on those areas first.

2. The book has many more exercises. Pick some exercises and try to do them. If you get stuck, explore a simple concrete example.

3. Look at the feedback you got on **Exam 1**. Did you lose any points on clarity or style? How would you rewrite your proof to be more clear?

The main goal of algebra is to prove things abstractly using symbols to represent concrete objects. However, to understand algebra well, it is extremely helpful to look at concrete examples. Your book has many.

Practice Problems

Abridged solutions to these practice problems will be posted later this week.

1. Let $H = \{\pm 1, \pm i\} \leq \mathbb{C}^\times$.

   (a) Prove that H is normal in $\mathbb{C}^\times$.

   (b) Describe explicitly the cosets of H.

   (c) Identify the quotient group $\mathbb{C}^\times / H$. (*Hint:* If you're stuck, first play around with the map $\psi : S^1 \times S^1$ given by $e^{i\theta} \mapsto (e^{i\theta})^2$.)

2. Let $\varphi : G \to G'$ be a surjective homomorphism between finite groups. Suppose $H \leq G$ and $H' \leq G'$ correspond to each other under the bijection in the Correspondence Theorem. Prove that $[G : H] = [G' : H']$.

3. Let q be a 5-cycle in S_n, where $n \geq 6$.

   (a) What is the cycle type of q^{17}?

   (b) In terms of n, how many permutations are there such that $pqp^{-1} = q$?

4. Prove that the conjugacy classes of a free group F_S (where S is the set of generators) are in bijection with the set of **closed words**, i.e. words that are written in a circle:
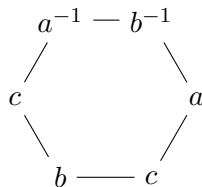
Exam Policies

The next page shows a mock version of the front of the exam packet. Read over the exam policies. Note that point values may not be accurate, and there will be small differences on the actual exam.

MAT 150A Fall 2023
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Exam 2

By providing my signature below I acknowledge that I abide by the University's academic
honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____   Name (print): _____

Name of left neighbor: _____   Name of right neighbor: _____

If you are next to the wall, then write "Wally" as your left or right neighbor. Write "Nemo" for
your left/right neighbor if you don't have a left/right neighbor, respectively.

Question	Points	Score
Q1	30	
Q2	40	
Q3	30	
Total:	100	

- This is a **closed-book** exam. You may not use the textbook, cheat sheets, notes, or any other
  outside material. No calculators, computers, phones, or any other electronics are allowed.

- The last page of the exam packet is provided for scratchwork. **Do not detach** this sheet
  from your exam packet.

- You have **45 minutes** to complete this exam. If you are done early, you may leave after
  handing in your exam packet.

- Everyone must work on their own exam. Any suspicions of collaboration, copying, or otherwise
  violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

- **This is a proof-based course.** All statements must be justified and argued in the style
  of a mathematical proof. Failure to do so will result in the loss of correctness and/or style
  points.