MAT 150A HW00 (Optional, not graded)

Solutions

Due Tuesday, 10/3/23 at 11:59 pm on Gradescope

Instructions Create a free Overleaf account, and create a new project (e.g. "MAT150A-hw"). Upload the provided `HW00.tex` file for HW00, and click on it on the sidebar. Then, press the big green "compile" or "recompile" button at the top of the right half of your screen. You should then see the HW00 PDF show up on the right half of your screen.

There are two exercises below. Type your solutions directly into the code, compiling every once in a while to admire your work. When you are done, press the download icon next to the recompile button to download your solutions as a PDF. Submit this PDF to Gradescope.

Reminder. *Your homework submission must be typed up in full sentences, with proper mathematical formatting. The following resources may be useful as you learn to use TeX and Overleaf:*

- *Overleaf's introduction to LaTeX:*
  https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes
- *Detexify:*
  https://detexify.kirelabs.org/classify.html

Exercise 1

Here's an example of how I would type up a formula in LaTeX:

Theorem. (Pythagorean Theorem) Let $a, b, c > 0$ denote the side lengths of a right triangle, where c is the length of the hypotenuse. Then

$$a^2 + b^2 = c^2.$$

Use TeX to typeset the quadratic formula. *Remember to define your variables and write in full sentences.*

SOLUTION.

Let $a, b, c \in \mathbb{R}$. The solutions to the quadratic equation $ax^2 + bx + c = 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Exercise 2

We often work with matrices, to *represent* group elements. Consider the following matrices:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

Compute $AB - BA$. *Copy-paste is your friend. Remember to write in full sentences.*

SOLUTION.

Using matrix multiplication, we compute:

$$\begin{aligned} AB - BA &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

Since $AB - BA \neq 0$, we see that $AB \neq BA$, i.e. $GL_2(\mathbb{R})$ (and also $GL_2(\mathbb{C})$) is non-abelian.