MAT 150A HW06

[add your name here]

Due Tuesday, 11/21/23 at 11:59 pm on Gradescope

Reminder. *Your homework submission* ***must be typed*** *(TeX'ed) up in full sentences, with proper mathematical formatting.*

Covered in this HW Chapter 6: isometries of the plane, symmetries of plane figures, lattices; translations, point groups, crystallographic restriction

Grading Some of the (parts of) problems will be graded in detail out of several points, and necessary feedback will be given. The rest will be graded out of 2 points.

Exercise 1

Prove that a conjugate of a glide reflection in $\mathrm{Isom}(\mathbb{R}^2)$ is also a glide reflection, and that the glide vectors have the same length.

SOLUTION.

Exercise 2

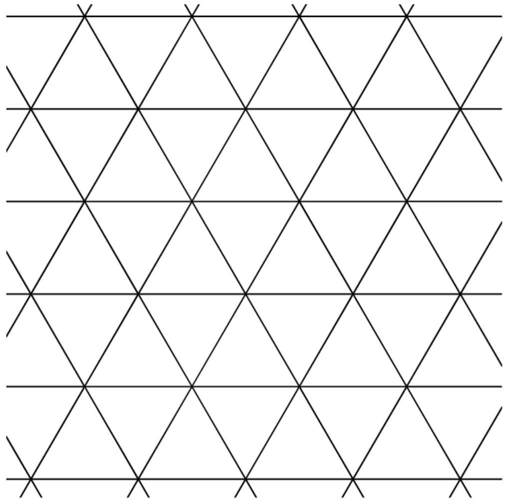
We used $\mathbb{R}^2$ to describe the points on the plane. We could equivalently use $\mathbb{C}$, the complex plane. Since we use the same notion of distance for points in the complex plane, as metric spaces, $\mathbb{R}^2$ is the same as $\mathbb{C}$. The generators of $\mathrm{Isom}(\mathbb{R}^2) = \mathrm{Isom}(\mathbb{C})$ on page 160, Equation (6.3.1), in the textbook, or slide 10 in Lecture 17.

Write formulas for the generators of $\mathrm{Isom}(\mathbb{C})$ in terms of the complex variable $z = x + iy$.

SOLUTION.

Exercise 3

Let G denote the group of symmetries of the following finite wallpaper pattern constructed from equilateral triangles of side length 1:



Source: https://mathworld.wolfram.com/TriangularGrid.html

(a) Determine the point group $\overline{G}$ of G, and find the index in G of the subgroup of translations L.

(b) Find translation vectors $a, b \in \mathbb{R}^2$ realizing L as the lattice $\mathbb{Z}a + \mathbb{Z}b$.

SOLUTION.