Lecture 08

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MAT 150A

Participation Slip

1. Take a slip from the front of the room.
2. Write your full name on the top left corner.
3. You will write down your answer to some clearly marked "Participation Slip" questions during lecture.
4. Hand in your slip at the end of class.

Reminder

- Participation will be graded starting **next lecture** (Lecture 9).
- A score of 15 (out of 20 lecture days) will receive full credit.

Recall: Equivalence relations, Partitions, and Cosets

An **equivalence relation** on a set S is a relation $\sim$ on elements of S that is

1. reflexive: For all a, $a \sim a$.
2. symmetric: If $a \sim b$, then $b \sim a$.
3. transitive: If $a \sim b$ and $b \sim c$, then $a \sim c$.

A **partition** P of a set S is a subdivision of S into nonoverlapping, nonempty sets.

Equivalence relations defined by maps

A set map $f : S \to T$ defines an equivalence relation on S, indexed by the elements of the image of f, $\text{im}(f) \subset T$:

$$P_t = f^{-1}(t).$$

- Here $f^{-1}(t)$ is the **inverse image** or **preimage** of $t \in T$.
- If $t \notin \text{im}(f)$, then $f^{-1}(t) = \emptyset$.
- We sometimes also say $f^{-1}(t)$ is the *fiber* of f over $t \in T$.

Warning: Here f^{-1} is symbolic notation. If f is not bijective, there is no inverse *function* f^{-1}.

Kernels are Normal

Recall

A subgroup N of G is **normal** if it is closed under conjugation by elements of G:

$$N \trianglelefteq G \qquad \text{iff} \qquad gng^{-1} \in N \text{ for all } g \in G.$$

Proposition

Let $\varphi : G \to G'$ be a homomorphism. Then the kernel $K = \ker \varphi$ is a normal subgroup.

Recall: Cycle Notation

Cycle notation

In S_5,

$$p = (1\ 3\ 4)(2\ 5)$$

is the bijective set map $p : [5] \to [5]$ given by the following chart:

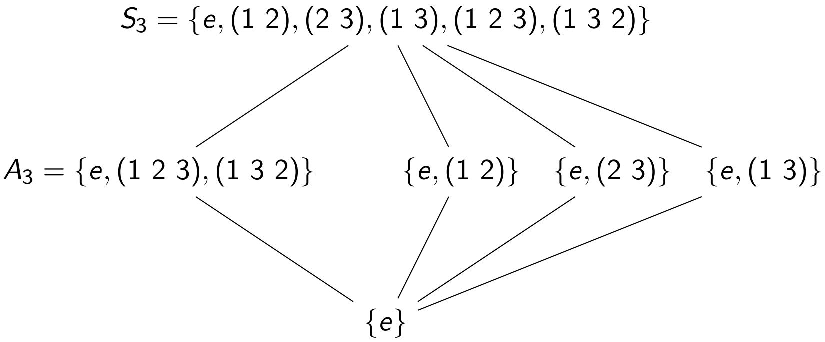
i	1	2	3	4	5
p(i)	3	5	4	1	2

There was some confusion about cycle notation conventions. Upon further inspection, my convention does in fact agree with the book's.

Re-definition

Any 2-cycle in S_n is called a **transposition.**

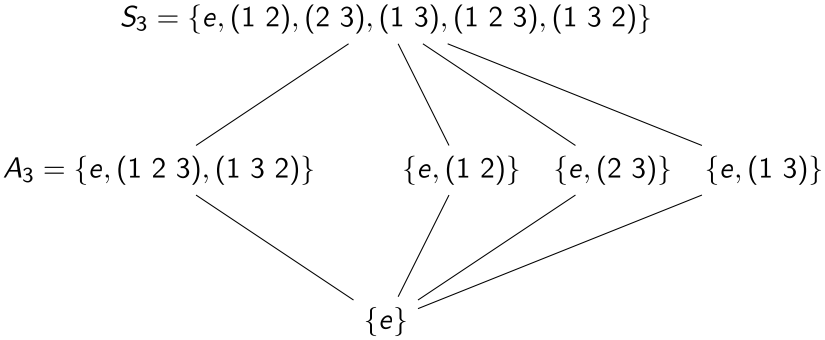
Example: Subgroup Structure of S_3



Participation Slip

1. Prove that A_3 is a normal subgroup.
2. Prove that $\{e, (1\ 2)\}$ is **not** a normal subgroup.

Example: Subgroup Structure of S_3



Question

What are the left cosets of $H = \{e, (1\ 2)\}$?

Generators and Relations (a first pass)

We can define **some** groups by giving a set of generators and a set of relations the generators satisfy:

$$G = \langle \{\text{generators}\} \mid \{\text{relations}\} \rangle = \langle g_1, g_2, \ldots, g_n \mid r_1, r_2, \ldots, r_k \rangle.$$

Each relation r is a **word** in the generators, indicating that we enforce the equation $r = 1$.

Let N be the largest normal subgroup of the **free group** $F = \langle g_1, g_2, \ldots, g_n \rangle$ generated by all the relations r_i.

Then $G \cong F/N$.

We will study quotient groups more carefully later.