Lecture 12

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MAT 150A

Participation Slip

1. Take a slip from the front of the room.
2. Write your full name on the top left corner.
3. You will write down your answer to some clearly marked "Participation Slip" questions during lecture.
4. Hand in your slip at the end of class, in the pile according to the first letter of your surname.

Reminders

- Exam 1 Wednesday; study guide is on class website.
- Today: Partial review, via the study of right cosets, exercises.
- HW03 due Tuesday at 11:59pm. You **must** typeset your homework with TeX.
- HW04 will be short, and will be posted after Exam 1.

Illness

If you are feeling sick or are otherwise unable to take the exam, you have the option to skip the exam. In this case, your Exam 1 grade will be replaced by your Final Exam grade at the end of the quarter.

If you still want to take the exam, please wear a mask when you come to class and sit far from other students.

Right Cosets

- The **right cosets** of a subgroup $H \leq G$ are the sets

$$Ha = \{ha \mid h \in H\}.$$

- They are equivalence classes for the equivalence relation

$$a \equiv b \quad \text{if} \quad b = ha, \text{ for some } h \in H.$$

and therefore form a partition of G.

These partitions aren't necessarily the same as the left cosets of G.

Example: S_3

$$S_3 = \{e, (1\ 2), (2\ 3), (1\ 3), (1\ 2\ 3), (1\ 3\ 2)\}$$

Exercise

Let $H = \{e, (1\ 2)\}$.

- What are the left cosets of H?
- What are the right cosets of H?

Exercise

Let $H \leq G$ and $g \in G$. Prove that $|gH| = |Hg|$.

Exercise

Prove that the number of left cosets of $H \leq G$ is equal to the number of right cosets of H.

Therefore we may define the index of H in G to be either the number of left cosets or the number of right cosets.

Proposition 2.8.17

Let $H \leq G$. The following conditions are equivalent:

1. $H \trianglelefteq G$.
2. For all $g \in G$, $gHg^{-1} = H$.
3. For all $g \in G$, $gH = Hg$.
4. Every left coset of H is a right coset.

Exercise

Prove Proposition 2.8.17.

Proposition 2.8.18

(a) If $H \leq G$, and $g \in G$, then gHg^{-1} is also a subgroup of G.

(b) If G has just one subgroup H of a particular order r, then that subgroup is normal.

Exercise

Prove Proposition 2.8.18.

Participation slip

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Write down a concept or topic you feel you may need more practice with.