Lecture 20

* fix lecture 19 slides

Reminders: ① Exam 2 Wednesday

practice + solutions are online; harder than exam problems. Make sure you know the main defns/theorems!

Recall from last time:

Plane $\mathbb{R}^2$ ↺ $\mathrm{Isom}(\mathbb{R}^2)$ Rigid plane transformations

points/vectors — translations, rotations, reflections, glide reflection (action of $O(2)$ on translation)

plane figures

discrete = points are isolated (ie $> \varepsilon$ apart)

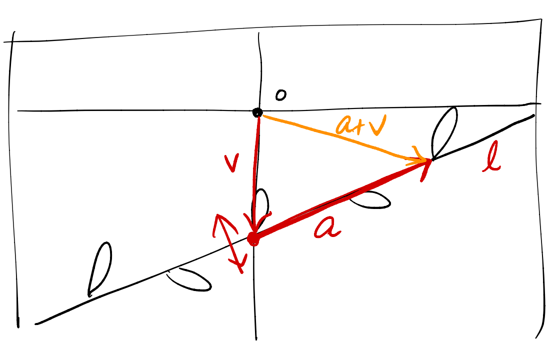
discrete = image of pt is $> \varepsilon$ away from pt.

Scenario: Imagine $G = \mathrm{Sym}(\Delta) \leq \mathrm{Isom}(\mathbb{R}^2)$, G is discrete

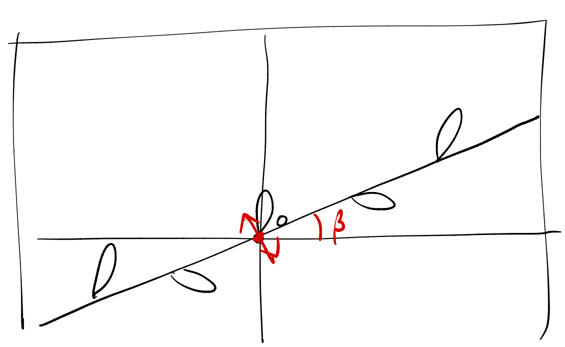
Translations: $\{\Lambda \leq \mathbb{R}^2\} \longleftrightarrow \{L \leq \mathrm{Isom}(\mathbb{R}^2) \mid L \text{ consists only of translations}\}$

Today the point group $\bar{G} \leq O(2)$ and $\bar{G} \curvearrowright L$

Q. How do we capture this glide reflection in terms ρ_θ, τ, t_a?



g



$\bar{g}$

first study this $\in O(2)$

The Point Group

(r in book; r = rueli)
τ = reflect across x axis

<u>Recall</u> $\pi : \text{Isom}(\mathbb{R}^2) \twoheadrightarrow O(2)$ $\quad$ $\ker \pi = T$, the subgroup of translations

<u>defn</u> For a discrete subgroup $G \leq \text{Isom}(\mathbb{R}^2)$,

the <u>point group $\overline{G}$</u> is $\pi(G) \leq O(2)$

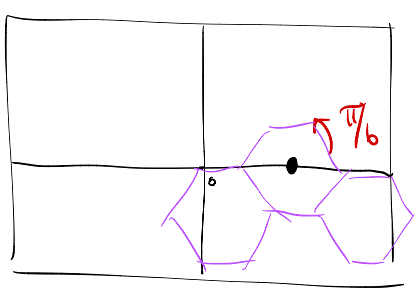
i.e. you can restrict $\pi|_G : G \to O(2)$, $\overline{G} = \text{im}(\pi|_G)$.

<u>notation</u>: For $g \in G$, let $\overline{g} = \pi(g) \in \overline{G}$ denote the orthogonal part

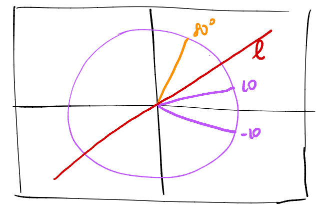
eg. Take the rotation ρ by $\pi/6$ around the point $(1,0)$: $t_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \rho_{\pi/3}$

Then $\overline{\rho} = \rho_{\pi/3} \in O(2)$.

$\pi^{-1}(\rho_{\pi/6}) = \{$ rotation by $\pi/6$ around all pts in $\mathbb{R}^2 \}$



eg. Let ℓ denote the line of reflection of $\rho_\theta \tau$



Observe: $\measuredangle$ of ℓ is $\frac{1}{2}\theta$.

Spend some time on this.

Pf. Consider the point $\approx re^{i\alpha}$.

$\tau(re^{i\alpha}) = re^{-i\alpha}$.

$\rho_\theta(re^{-i\alpha}) = re^{-i\alpha} \cdot e^{i\theta} = re^{i(\theta-\alpha)}$

Gets to idea of crystallographic

To reflect $re^{i\alpha} \leftrightarrow re^{i(\theta-\alpha)}$, we reflect across the line containing the ray $\measuredangle = \theta/2$.

$\bar{G}$ contains $\rho_\theta \tau$ if there is an element $t_a \rho_\theta \tau$ in G.