Lecture 1

Syllabus + Class overview

- Webpage, syllabus, book, class calendar
- OH start next week

What is a matrix?

<u>defn 1</u> (Abstract) A matrix is a 2D array of values (scalars).

eg. $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$

eg. ATELs = astronomy telegrams

<u>Term-document matrix:</u>

$$\begin{array}{c|c} & \overbrace{\begin{matrix} \text{ATEL1} & \text{ATEL2} & \text{ATEL3} & \cdots \end{matrix}}^{\text{documents}} \\ \text{terms}\left\{\begin{matrix} \text{"star"} \\ \text{"nebula"} \\ \text{"quasar"} \\ \vdots \end{matrix}\right. & \begin{bmatrix} 3 & 0 & \boxed{\begin{matrix} 1 \\ 0 \\ 0 \\ \vdots \end{matrix}} \\ 0 & 2 & \\ 1 & 0 & \end{bmatrix} \end{array}$$

← vector for 3rd document

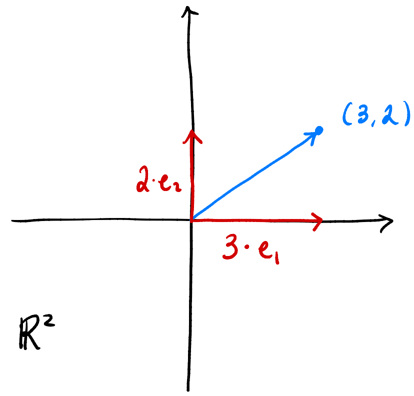
<u>defn 2</u> (List of vectors)

A matrix is a list of <u>vectors</u> of the same length

eg. $\left\{ \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\} \rightsquigarrow \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$

__Aside__ geometric interpretation of vectors

eg.



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} = 3 \cdot e_1 + 2 \cdot e_2$$

- $\{e_1, e_2\}$ is a __basis__ for the __vector space__ $\mathbb{R}^2$ because any vector $\vec{v}$ in $\mathbb{R}^2$ can be written as

$$\vec{v} = c_1 \cdot e_1 + c_2 \cdot e_2$$

__defn 3__ (linear transformation)

A matrix is a __linear transformation__ between two vector spaces that have __chosen bases__.

eg. Consider $\mathbb{R}^2$ with the standard bases $\{e_1, e_2\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

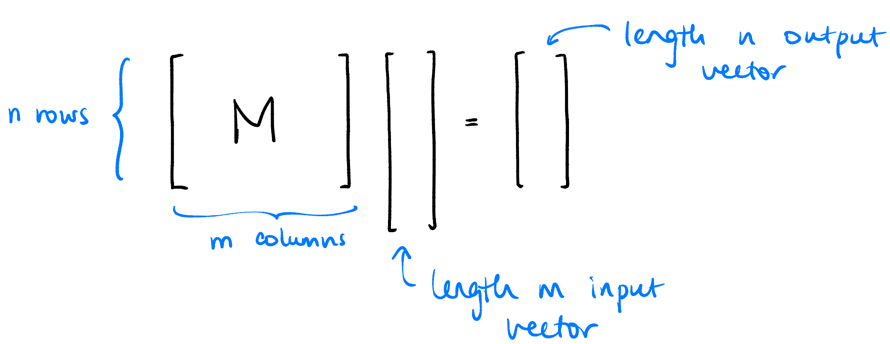
Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

$$A \cdot e_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

So $A \cdot e_1 = e_2$.

"A" represents a linear transformation $A : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$.

__Rmk.__ Dimensions of a matrix:



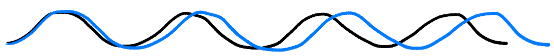
$\Rightarrow$ M is a linear transformation $\mathbb{R}^m \longrightarrow \mathbb{R}^n$.

Bases are important

eg. "Beating" or "Beats" in music

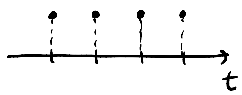
Guitar 1: 

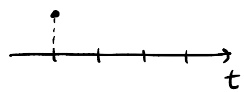
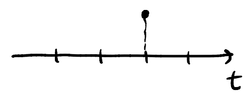
Guitar 2: 

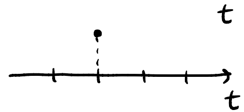
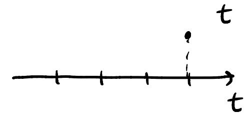


will eventually sync up again.

Now suppose I want to send you the signal



<u>Naïve basis</u>: e_1 =  e_3 = 

e_2 =  e_4 = 

$\text{signal} = e_1 + e_2 + e_3 + e_4$ (4 coefficients needed)

<u>Better basis</u>: use sine waves to approximate!

u_1 = guitar 1, u_2 = guitar 2

$\text{Signal} \approx u_1 + u_2$ (2 coefficients needed)

* This is the basic idea behind <u>audio</u> <u>compression</u>.