LECTURE 2

- MATLAB access info on website
- Download text while on campus

Q. $\underbrace{\begin{bmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}}_{M} \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$

As a linear transformation, what are the domain and codomain of M?

Notations & Conventions

* Generally, $[n] = \{1, \dots, n\}$. (Sometimes, a 1×1 matrix, but rarely)

$\mathbb{R}$ = real #s

$\mathbb{R}^d = \underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{d \text{ copies}}$

(column) vector: $\vec{v} \in \mathbb{R}^d$ $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_d \end{bmatrix}$

or bolded when typeset

transpose: $\vec{v}^T$ ($\in \mathbb{R}^d$, a dual one)

$= \begin{bmatrix} v_1 & \cdots & v_d \end{bmatrix}$

matrix $A \in \mathbb{R}^{n \times m}$

$n \left\{ \underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & \ddots & & \\ \vdots & & & \\ a_{n1} & \cdots & & a_{nm} \end{bmatrix}}_{m} \right.$

transpose: $A^T \in \mathbb{R}^{m \times n}$

$\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & & & \vdots \\ \vdots & & \ddots & \\ a_{1m} & \cdots & & a_{nm} \end{bmatrix}$ where $A = (a_{ij})$

- the j^{th} column of A: $a_{\bullet j}$ $(1 \le j \le m)$ $j \in [m]$ $a_{\bullet j} \in \mathbb{R}^n$
- the ith row of A: $a_{i \bullet}$ $(1 \le i \le n)$ $i \in [n]$ $a_{i\bullet} \in \mathbb{R}^m$

<u>Scalar multiplication</u>: If $A = (a_{ij})$, $c \cdot A = (c \cdot a_{ij})$ (where $c \in \mathbb{R}$)

Q. If A is a term-document matrix, what do the following represent?

① a_{ij} ② $a_{i\bullet}$ ③ $a_{\bullet j}$

<u>Standard</u> or <u>canonical</u> basis vectors for $\mathbb{R}^d$:

$e_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ ← ith row

eg. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 3 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = e_1 + 2e_2 + 3e_3$

<u>Identity matrix</u> $I = I_k \in \mathbb{R}^{k \times k}$ $\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$

ie. $a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{o/w} \end{cases}$

Q. What is $c \cdot I_k$?

(Chapter 2)

Matrix - vector multiplication

I'm using new variables here! Learn the algorithm, don't memorize the symbols.

① Symbolically: Let $A \in \mathbb{R}^{m\times n}$

Q. domain, codomain?

$m\{\underbrace{[\quad]}_{n} \Rightarrow \mathbb{R}^n \xrightarrow{A} \mathbb{R}^m$

Let $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ (variables that represent vectors)

Then $y = Ax$ means $y_i = \sum_{j=1}^{n} a_{ij} x_j$ $(1 \le i \le m)$

eg. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

② Pseudo code

algorithm matrix-vector-multiplication is:

input: matrix A(i, j) for i in [m], j in [n]
vector x(j) for j ∈ [n]

output: vector y_i

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for i in 1, ..., m :
    define y(i) = 0
    for j in 1, ..., n,
        y(i) = y(i) + A(i,j) * x(j)
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* each operation here is a "flop" = floating point operation

③ Meaning: a "linear" transformation

"Linearity" A function $f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is ($\mathbb{R}$-) linear if

$\forall\ c \in \mathbb{R},\ x \in \mathbb{R}^n,\quad f(c \cdot x) = c \cdot f(x)$

Intuition / visually: Shear, stretch, reflect
no translation or twisting.