LECTURE 4

Q. D_1 = " The inner product of vectors defines a norm on vectors."

D_2 = " The outer product of two vectors is a matrix."

D_3 = " Any norm defines a distance function."

(a) Write down the term document matrix. (Answers will vary.)

(b) Query vector for " vector norm" q_1

"vector product" q_2

" matrix norm" q_3

defn. A vector norm is a mapping $\|\cdot\| : \mathbb{R}^n \longrightarrow \mathbb{R}$ such that:

- $\|x\| \geq 0 \ \forall x$, $\|x\| = 0$ iff $x = 0$
- $\|\alpha x\| = |\alpha| \, \|x\|$, $\alpha \in \mathbb{R}$
- $\|x+y\| \leq \|x\| + \|y\|$ the triangle inequality

Given any vector norm, we can define a distance function

$$d : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$$
$$(x, y) \longmapsto d(x,y) = \|x - y\|$$

Approximations of vectors: $\bar{x}$ approximates x.

The absolute error is $\|\delta x\| = \|\bar{x} - x\|$

The relative error is $\frac{\|\delta x\|}{\|x\|} = \frac{\|\bar{x} - x\|}{\|x\|}$ (if $x \neq 0$)

eg. Survey of time ① Waiting for ambulance to arrive

② Waiting for the bus in Boston.

* Note this is about 1D vectors; emphasis on error
(eg. of survey average vs. actual value)

Fact In any finite-dimensional vector space, all vector norms are quite comparable, in the following sense:

For any two norms $\|\cdot\|_\alpha, \|\cdot\|_\beta : \mathbb{R}^n \longrightarrow \mathbb{R}$, $\exists$ consts m, M s.t.

$\forall x$, $m\|x\|_\alpha \le \|x\|_\beta \le M\|x\|_\alpha$. (m, M indep of x.)

This implies that if $(x_i)_{i=1}^{\infty} \longrightarrow x^*$, then

$\lim_{i\to\infty} \|x_i - x^*\| = 0$ for any $\|\cdot\|$.