MAT 167
Instructor: Melissa Zhang
Midterm Exam

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Name of left neighbor: _____ Name of right neighbor: _____

If you are next to the wall, then write "Wally" as your left or right neighbor.

Problem	Points	Score
Q1	30	
Q2	40	
Q3	30	
Total:	100	

- This is a **closed-book** exam. You may not use the textbook, cheat sheets, notes, or any other outside material. No calculators, computers, phones, or any other electronics are allowed. The purpose of this exam is to test your basic understanding of the material.
- The last page of the exam packet is provided for scratchwork. **Do not detach** this sheet from your exam packet.
- You have **45 minutes** to complete this exam. If you are done early, you may leave after handing in your exam packet.
- Everyone must work on their own exam. Any suspicions of collaboration, copying, or otherwise violating the Student Code of Conduct will be forwraded to the Student Judicial Board.
- Read each problem carefully, and write down every step of your reasoning clearly. Your solution must be **neat**; if we can't read, can't follow, or can't find your solution, you will not receive credit.

1. Suppose we have the following items on a coffee shop menu:

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~1~ Iced Caramel Latte
~2~ Iced Decaf Americano
~3~ Vanilla Soy Latte
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   (a) The *documents* D_1, D_2, D_3 are already listed in a particular order above. Write down all the *terms* in alphabetical order. This fixes a basis for both the domain and codomain of our term-document matrix. Then, viewing the term-document matrix A as a linear transformation

$$A : \mathbb{R}^{\#\text{documents}} \to \mathbb{R}^{\#\text{terms}},$$

   write down the matrix A.

   (b) Write down the query vector $\mathbf{q}$ for the query "`Iced Latte`".

(c) Compute $\cos\theta(D_1, \mathbf{q})$, the *cosine of the angle* between D_1 and the query vector.

2. Consider the matrices and vectors

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 3 \end{bmatrix} \qquad \mathbf{p} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \qquad \mathbf{q} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad \mathbf{r} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

(a) Find the L_1-norms $\|a_{\bullet 1}\|_1$ and $\|a_{\bullet 2}\|_1$.

(b) Compute $\mathbf{p}\mathbf{q}^\top$ and find its rank.

(c) Which of the vectors $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are in the column span of A? Justify your answer.

(d) Compute the Frobenius norm $\|A\|_F$. (*You may leave your answer as an expression that can be directly entered into a scientific calculator, i.e. you may use* $+, -, \times, \div$, *and powers.*)

(e) Compute the L_2-operator norm $\|A\|_2$.

3. We would like to find the best-fit line $y = c_0 + c_1 x$ through the three points $(x, y) = (0, 1), (1, 1)$, and $(2, 4)$, using the least-squares method.

   (a) Write down the overdetermined system in the matrix-vector form

$$A\mathbf{c} = \mathbf{y} \qquad \text{where} \qquad \mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}.$$

   (b) Write down the normal equations, and then compute the best-fit line by solving the normal equations. (Your final answer should be an explicit equation for the best-fit line.)

Scratchwork

Nothing on this page will be graded.