Lecture 1

Syllabus + Class overview

- Webpage, syllabus, book, class calendar
- OH start next week

Knots + Topology

<u>defn 1</u> A <u>knot</u> is a loop in 3D space $\mathbb{R}^3$.

Knot diagram:

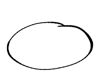
unknot — right-handed trefoil — also the unknot

- projection onto the plane + <u>crossing</u> information

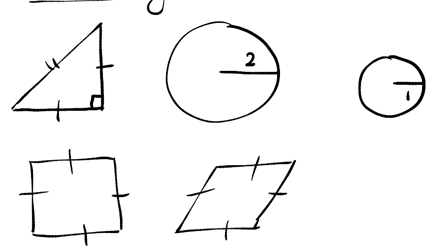
<u>Topologist's worldview</u>:

Topology and geometry both care about shapes. Geometry cares about lengths and angles. Topology only cares about the fundamental properties of the shape.

<u>Topology</u>:

 U = unknot

<u>Geometry</u>:



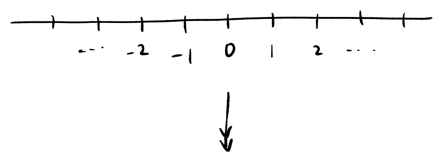
All these are the same — as unknot

Reidemeister Moves

<u>defn 2</u> A <u>knot</u> is an equivalence class of knot diagrams modulo <u>planar isotopy</u> and <u>Reidemeister moves</u>.

<u>Aside</u>: Equivalence relations & equiv classes

eg. $\mathbb{Z}$ =

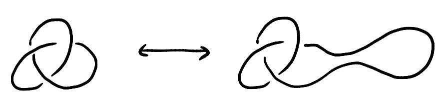
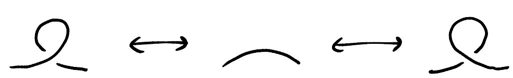
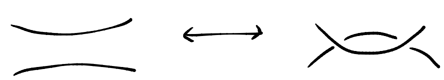


For $m, n \in \mathbb{Z}$,

$m \sim n$ iff $m \equiv n \pmod 2$.

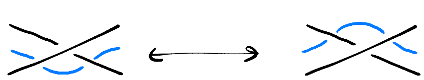
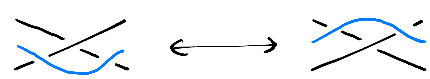
$\mathbb{Z}/2\mathbb{Z}$ $\{\text{even}, \text{odd}\}$

<u>thm</u> (Reidemeister) Given a knot K and two diagrams D_1, D_2 representing it, D_1 and D_2 are related by (finitely many) of the following moves:

- planar isotopy 
- Reidemeister 1 (R1): 
- Reidemeister 2 (R2): 

  * These "local" pictures can be rotated by any angle

- Reidemeister 3 (R3):

 <u>OR</u> 

Knot Invariants

<u>defn.</u> A knot invariant is a labelling (a "functor")

$$\text{Knot diagrams} \longrightarrow \text{Some type of mathematical object}$$

such that

given diagrams D_1, D_2 of a knot K,
the label assigned to D_1 and D_2 are the <u>same</u>.

In otherwords, this labelling descends to a well-defined labelling

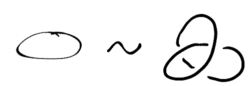
$$\text{Knots} = \text{Knot diagrams}/\sim \;\longrightarrow\; \text{Some type of mathematical object}$$

eg. "Crossing number"

Consider the assignment

$$\text{Knot diagrams} \longrightarrow \mathbb{N} \cup \{0\} = \mathbb{Z}_{\geq 0}$$

$$D \longmapsto \#\text{ of crossings in } D$$

This is <u>not</u> a knot invariant yet! eg. 

But we can tautologically define the following knot invariant:

<u>defn.</u> The <u>crossing number</u> of a knot K is

$$\min\{\#\text{ crossings in } D \mid D \text{ is a diagram for } K\}.$$

- As a labelling from knot diagrams $\to \mathbb{Z}_{\geq 0}$,

  $$\text{crossing number}(D) = \min\{\#\text{ crossings in } D' \mid D' \sim D\}.$$

- We use crossing number to organize our tables listing various knots. ("Tabulating knots")