Lecture 4

tangles, braids
tri colorability

Q. defn A knot is amphicheiral if $K \sim m(K)$ where $m(K)$ is the mirror knot

Show that the Figure 8 knot is amphicheiral.

How would you connect the idea of writhe to amphicheirality?

Link Invariant: Linking numbers

then do seifert surfaces

Given an oriented link (L, o)

Let (L, o) be an oriented link (presented as a diagram D)

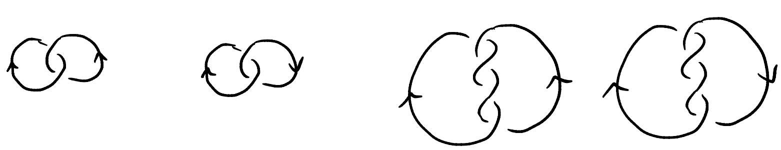
Let C_1, C_2 be two components of L.

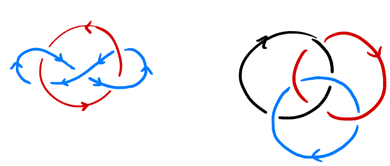
(By an abuse of notation, also their diagrams in D.)

The linking # b/w C_1 and C_2 is $\dfrac{\#\oplus - \#\ominus}{2}$,

where $\oplus, \ominus$ are only count crossings involving both C_1 and C_2.

eg. ① 

② 

③ 

Claim Reidemeister moves preserve linking #.

Pf. Check R1-R2-R3.

ex. Split vs. splittable link

Knot Invariant: Tricolorability

* we haven't actually proven that there are any knots that are the unknot! (I've been claiming so!)

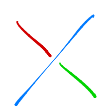
<u>defn</u> <u>Strand</u> = one continuous curve you draw in a knot diagram

A <u>tricoloring</u> of a knot diagram is an (surjective) assignment of colors to strands:

$$\left\{\begin{matrix}\text{strands in}\\ D\end{matrix}\right\} \longrightarrow \overset{(3\text{ colors})}{\{c_1, c_2, c_3\}}$$

such that at each crossing, either

(i) all are the same color or (ii) all three colors are used



and... at least two colors are used! (probably all three...)

eg.

U 3$_1$ 4$_1$ (HW02) 7$_4$

Key concept:

$$\begin{tikzcd}
\{\text{Knot diagrams}\} \arrow[r] \arrow[d, "/R1,2,3"'] & \{\text{tricolorable, not tricolorable}\} \\
\{\text{Knots}\} \arrow[ur, dashed, "?"'] &
\end{tikzcd}$$

* If we can show tricolorability is preserved under Reid moves, then we have a knot invariant, ie a well-defined map above.

<u>Claim</u> Tricolorability is preserved under Reid. moves.