LECTURE 5

Daping covering

Giedt Hall 1006
1:10 pm - 2:00 pm

Purple: Instructions for Daping.
Black: Actual lecture notes
Blue: Commentary (usually vocal)
Red: Emphasize

⊛ There is more than enough material for a class. After class, just let me know how far you got.

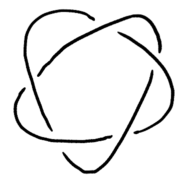
I. Review Problem

1. Place slips of scratch paper on front desk

⊛ Collect these + put them in my mailbox!

2. Write review problem on the board:

Q. Determine whether this knot is tricolorable:

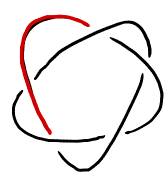


3. Give them 2-4 minutes to think & discuss with classmates.

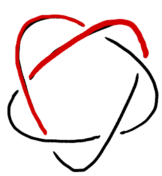
4. Solicit answers / Present solution (your choice)

A. No. Suppose it were. Then this diagram would have a tricoloring.

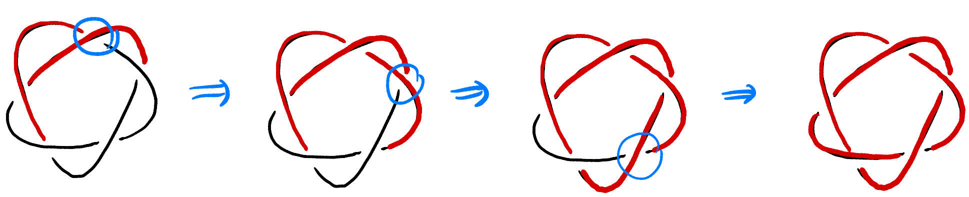
WLOG (without loss of generality),



If we did

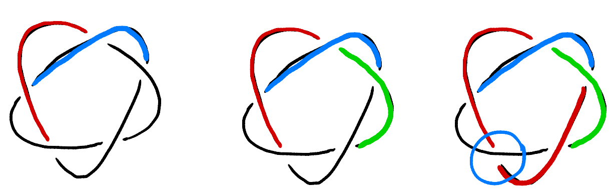


then



(contradiction).

Therefore we must have



(also a contradiction.)

//

II. Introduce yourself if you haven't already!

<u>Knots and Planar Graphs</u> (§2.4)

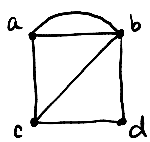
Graphs are ubiquitous bookkeeping tools in math, and are especially studied in <u>combinatorics</u>, which has a lot ties to knot theory.

<u>defn.</u> A graph is the following data:

- a <u>vertex set</u> V
- an <u>edge set</u> E consisting of (unordered) pairs of vertices in V (maybe with multiplicity)

(Some people want me to say "are" data. No, "data" is like "people". There <u>are</u> <u>people</u> who would refer to a particular group as <u>a people</u>.)

eg.



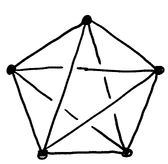
$V = \{a, b, c, d\}$

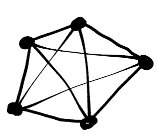
$E = \{ \{a,b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\} \}$

(Clearly easier to draw the diagram in many cases.)

The above graph is <u>planar</u> since it's embedded in the plane.

A nonplanar graph cannot be embedded in the plane. Eg:



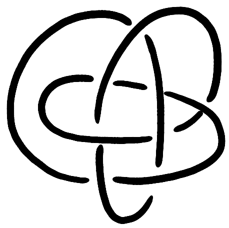
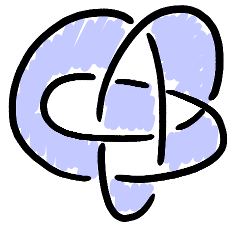
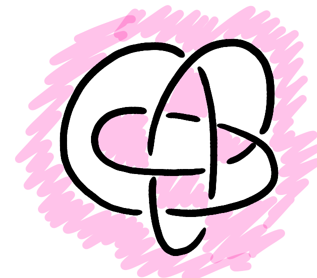


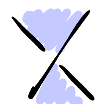
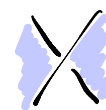
In these cases we can either

① Draw the graph with crossings, or

② Draw the actual projection (w/o crossing information) but make the vertices clear as large dots.

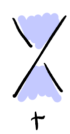
From knot (or link) diagram to a planar graph:

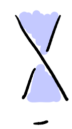
eg.   or 

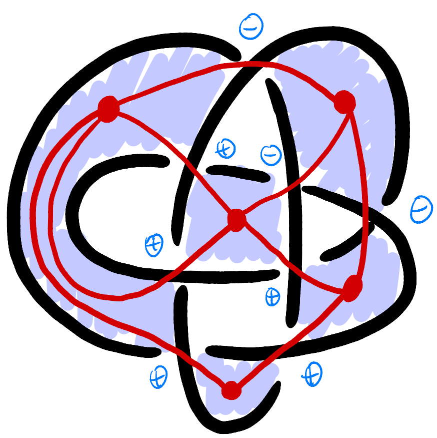
dfn A checkerboard coloring of D is an assignment of the regions of the plane cut up by the diagram to two colors, so that near any crossing, we have  or 

* Not to be confused with positive/negative signs for crossings in an oriented knot diagram!

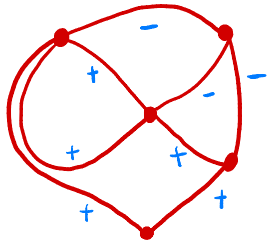
Signs:

+ 

− 

eg. 

Now assign the shaded regions to vertices, crossings to edges.

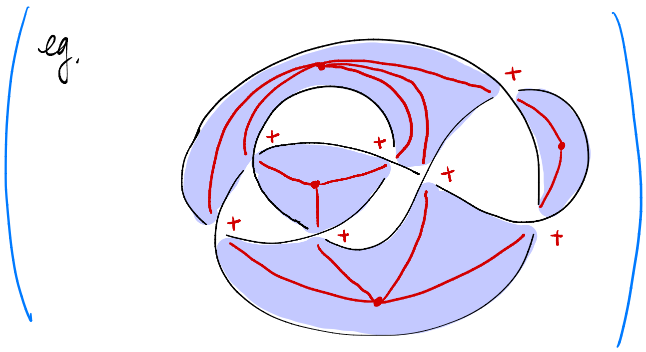


this signed planar graph contains all the data of the original knot diagram

Fact. If a diagram is alternating, then either of the associated signed planar graphs have only one type of sign. (HW03)

(This is on their next HW.)

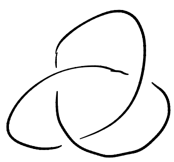
e.g.



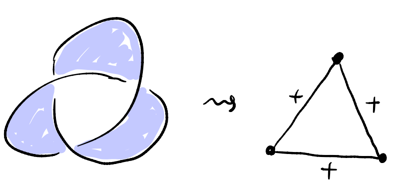
← skip this in class; you can use the following trefoil example instead

(Have the students try the following exercises if you have time)

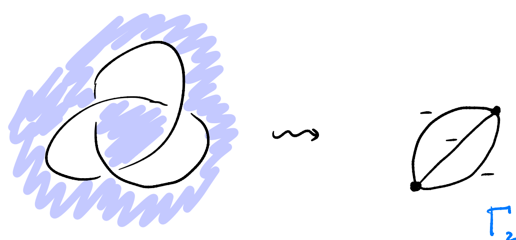
ex. Checkerboard color this diagram (you have two choices)



and draw the associated signed planar graph.



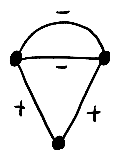
Γ_1



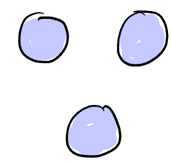
Γ_2

Q. What's the relationship b/w these two graphs?
(They are planar duals – we'll see more about the combinatorics of graphs next time).

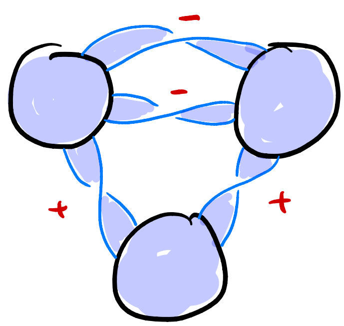
ex. Draw the knot diagram from this signed planar graph:



Answer: First draw filled-in circles ("disks") for the vertices:



Then add twisted bands connecting them, according to the signs.



Finally, take the boundary (outline) of the resulting surface: