LECTURE 9

Q. Show that for any knot K, $\Delta_K(t) = \Delta_K(t^{-1})$.

The Kauffman bracket – another approach to the Jones Polynomial

$\langle\, , \rangle$ = \langle, \rangle   ✱ local pictures!

Defined recursively: (Still start w/ oriented knot... we'll see why)

Book:

① $\langle \bigcirc \rangle = 1$

② $\langle \times \rangle = A \langle)(\rangle + A^{-1} \langle \asymp \rangle$

③ $\langle L \cup \bigcirc \rangle = (-A^2 - A^{-2}) \langle L \rangle$

Our convention (for later useful)
⚠ not knot invt yet

① $\langle \phi \rangle = 1$

② $\langle \times \rangle = \langle)(\rangle - q \langle)\,(\rangle$

③ $\langle L \cup \bigcirc \rangle = (q + q^{-1}) \langle L \rangle$

We were irresponsible with the other polynomials:

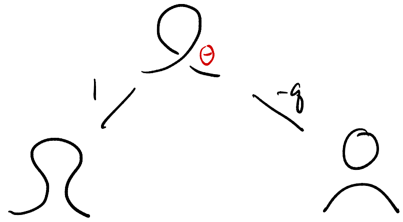
① Why does the resolving tree terminate?

② What happens under Reidemeister moves?

① Is much easier to see here; can always resolve down to unlinks; the full binary tree will be needed

② Let's look at Reid 2 and 3 first:

Reidemeister 1:



$$= (q+q^{-1})(-q)\langle \frown \rangle$$

$$\Rightarrow \langle \Omega \rangle = \langle \Omega \rangle - (q^2+1)\langle \frown \rangle = -q^2 \langle \frown \rangle$$

Try the other one:

$$\langle \Omega_{\oplus} \rangle = \langle \Omega \rangle - q\langle \Omega \rangle$$

$$= (q+q^{-1})\langle \frown \rangle - q\langle \frown \rangle$$

$$= q^{-1}\langle \frown \rangle$$

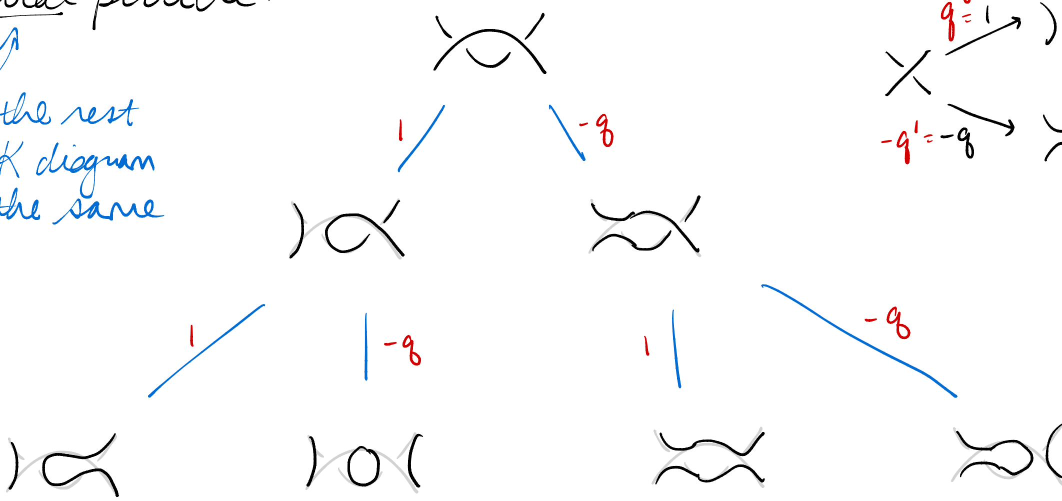
⇝ normalization:

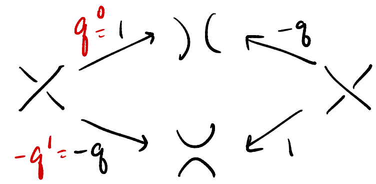
$$(-1)^{-n_-} q^{n_+ - 2n_-} \cdot \langle L \rangle = \tilde{V}_L(q)$$

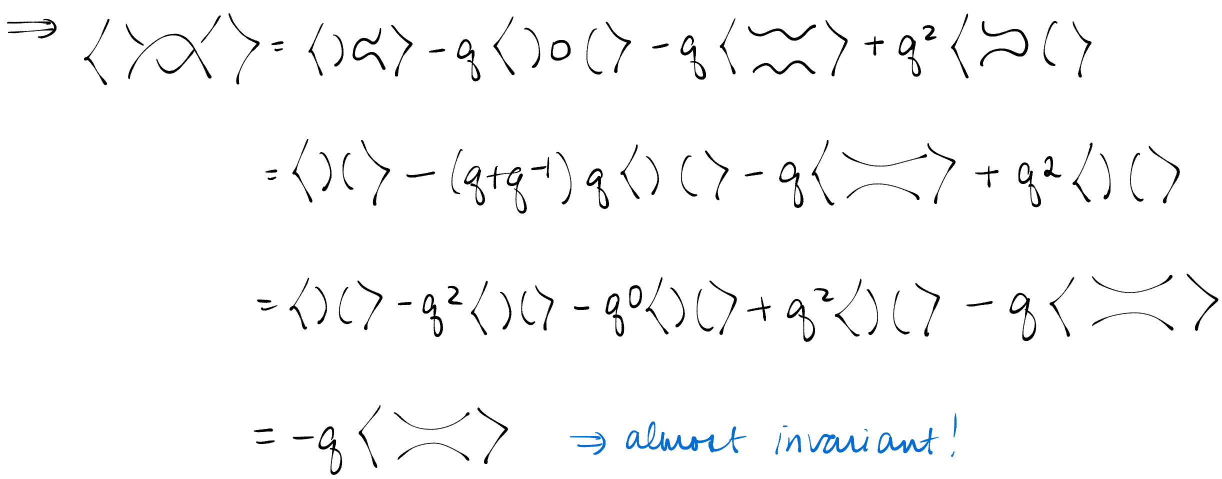
Then divide out by $q+q^{-1}$ (always a factor)

R2 local picture:

(imagine the rest of the K diagram staying the same)

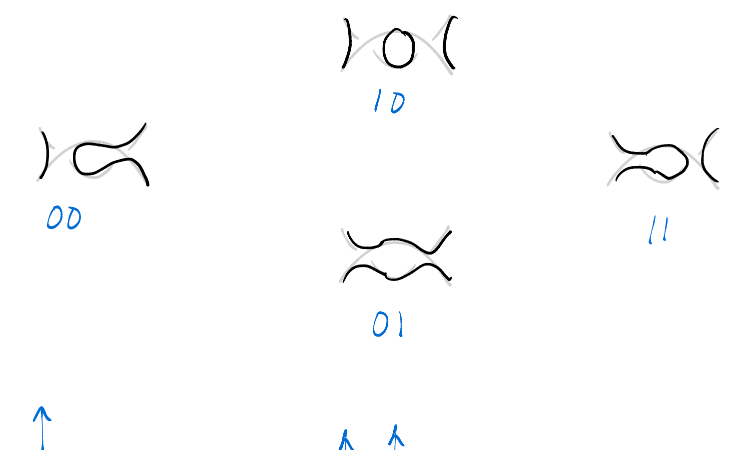






⇒ almost invariant!

Reorganize:



$1 \cdot \langle\ \rangle + (-q)\cdot\langle \text{sum of brackets} \rangle + (-q)^2 \langle \uparrow \rangle$

ie coeff for $\alpha \in \{0,1\}^2$, is $(-q)^{|\alpha|}$.