LECTURE 10

Chapter 4: Surfaces + Knots

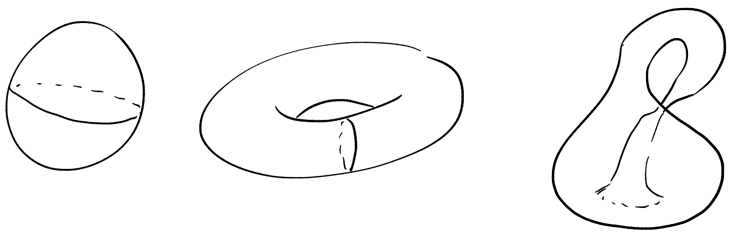
Q. What knot is the boundary of a Möbius band?

What happens if I twist the band 2x before gluing? K times?

defn. A (closed) surface or 2-manifold Σ is a topological space that locally looks like $\mathbb{R}^2$.

a neighborhood of $p \in \Sigma$ is a set of points close to it (to carefully define this, we need point-set topology)

eg.

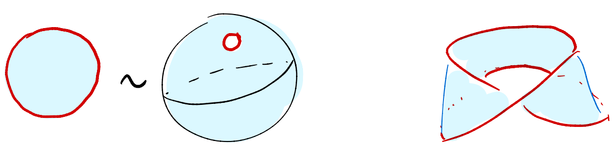


* Not embedded in any ambient space!

a surface with boundary $(F, \partial F)$ has 2 kinds of points:

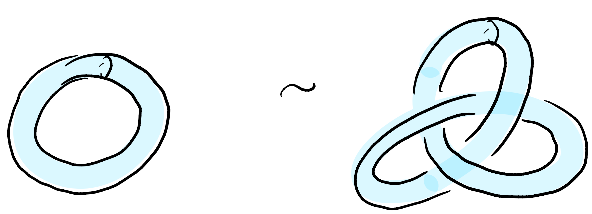
- $p \in \text{interior}(F) = \mathring{F} \rightsquigarrow \nu(p) \sim \mathbb{R}^2$
- $p \in \text{boundary}(F) = \partial F \rightsquigarrow \nu(p) \sim \mathbb{R} \times [0, \infty)$

eg.

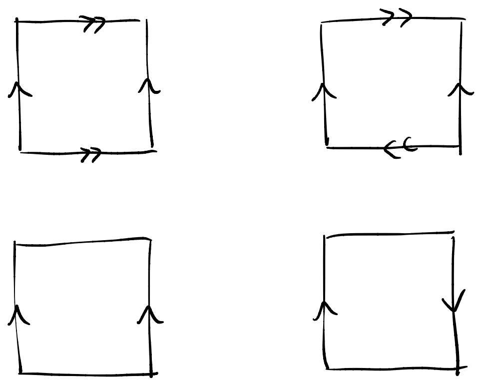


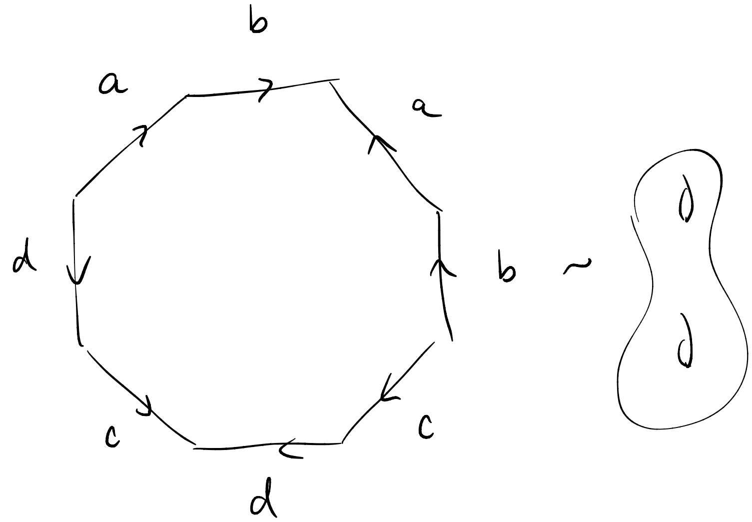
Surfaces are the "same" if they are homeomorphic, ie you can match the points of F and F' up so that the nbhd behavior matches too:

eg.



"Gluing" (Identification spaces)





Properties:

- Orientability: Torus vs Klein bottle
  Annulus vs. Mobius strip
  either $\exists$ or $\nexists$ an orientation reversing curve (either 1 or 2 sides)
- Genus – intuitively
- Euler characteristic (CW decomposition)
  - break down the surface into pieces that look like dots, lines, and disks glued together $\chi = V - E + F$
  - From , we see that for a closed orientable surface, $\chi = 2 - 2g$.

Fact χ is a surface (homeomorphism class) invariant!

Why? Consider ① different ways to cut
② different pictures (does this matter?)