LECTURE 11

Q. Draw a planar graph Γ. Compute

$$\chi = \#\text{vertices} - \#\text{edges} + \#\text{regions}$$

Why is this always 2?

<u>Surfaces, continued</u>

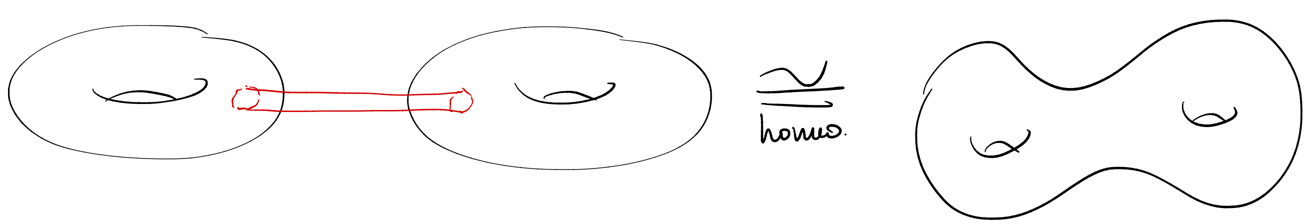
- What does "same" mean?

<u>Rough defn</u> Σ and Σ' are homeomorphic if there is a bijection (of sets) $f: \Sigma \to \Sigma'$ such that as you vary the point $x \in \Sigma$, the image $f(x) \in \Sigma'$ varies continuously.

(There is a continuous bijection $\Sigma \to \Sigma'$. We won't go further into this discussion of point-set topology; if you're interested, see Munkres' book.)

- Genus - intuitively # donut holes

<u>defn</u> # of surfaces:



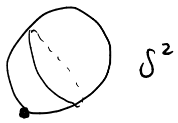
<u>Fact</u> (Classification of surfaces) Let Σ be a closed surface.

Then X is in one of these 2 families:

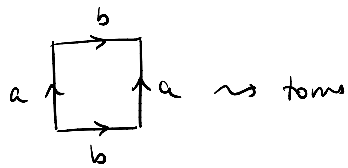
- Orientable: $\#^k$ Torus $\quad$ (S^3, torus, ...) $\quad$ $k \in \mathbb{N} \cup \{0\}$
- Nonorientable: $\#^k \mathbb{RP}^2$ $\quad$ $k \in \mathbb{N}$

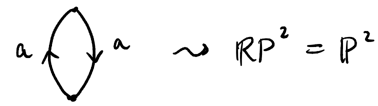
We can describe these as identification spaces! (Glued spaces)

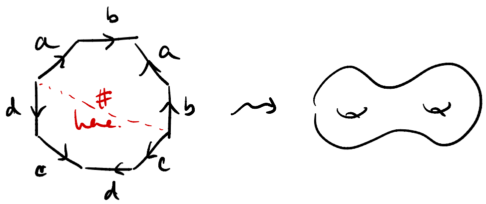
Fact

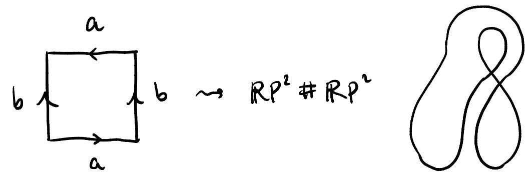


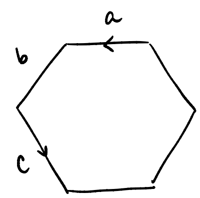
This is very cool stuff but we will be moving on to the knot theory!











...

Lets focus on orientable surfaces. We can tell them apart by their Euler characteristic:

$$\chi = \#\text{ vertices} - \#\text{ edges} + \#\text{ faces} \qquad = 2-2g.$$

Fact χ is independent of the construction!