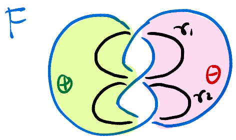
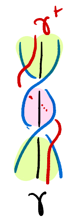
LECTURE 13

Q Consider the following Seifert surface for the right-handed trefoil and the curves γ_1 and γ_2 on the surface F:



Draw the curves γ_1^+ and γ_2^+, which are obtained by pushing γ_1, γ_2 a little bit off F in the $\oplus$ direction:

eg.



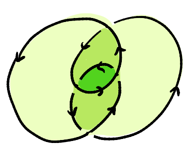
Last time: Seifert's algorithm for producing Seifert surfaces for K from diagrams D of K.

Q Why does the algorithm always work?

- If disks D, D' are on the same level & next to each other (ie connected by a crossing), then they obviously have opp orientations
- If on different but adjacent heights (ie nested) then clear they have the same orientation.

Q Do you always get min genus surfaces from this algorithm?

A. • In general, no:



$\chi = 3 - 4$

$\chi(F) = 3 - 4 + 1 = 0 = 2 - 2g$.

$\Rightarrow g(F) = 1$.

But this is unknot.

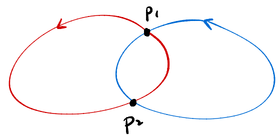
- Fact If the diagram is alternating, then yes!

(No proof given, but you could explore this as a final project.)

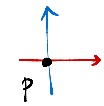
Toward the <u>linking matrix</u> or <u>Seifert form</u> of a Seifert surface F

<u>Aside</u> Signed intersection number (algebraic intersection number)

eg.



If we locally see



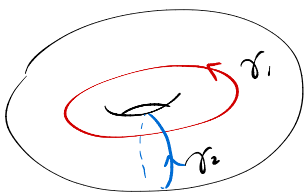
, the intersection of γ_1 (red) and γ_2 (blue) here is $+1$.

(We see γ_1 oriented as the positive x-axis and γ_2 is oriented as the positive y axis.)

In the example above, at p_1 we have a $+1$ intersection, while at p_2 we have a -1 intersection.

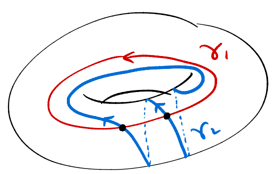
$\Rightarrow I(\gamma_1, \gamma_2) = +1 - 1 = 0$

eg.



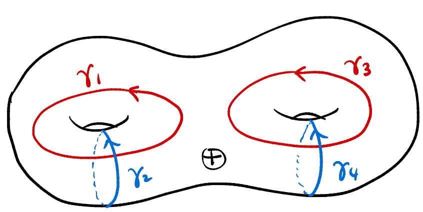
$I(\gamma_1, \gamma_2) = +1$ while $I(\gamma_2, \gamma_1) = -1$.

eg.



$I(\gamma_1, \gamma_2) = +2$, $I(\gamma_2, \gamma_1) = -2$.

Consider the closed genus 2 surface Σ_2, embedded in $\mathbb{R}^3$ as shown:



The linking matrix for this surface will be given in the form

	γ_1^+	γ_2^+	γ_3^+	γ_4^+
γ_1	0	-1	0	0
γ_2	0	0	0	0
γ_3	0	0	0	-1
γ_4	0	0	0	0

where the $(i,j)^{th}$ entry is $lk(\gamma_i, \gamma_j^+)$.

Here, γ_i^+ is γ_i, but pushed a little bit off Σ_2 in the positive direction.

Next time: I'll explain what $H_1(\Sigma_2)$ is, ie what the "basis" $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ is.