# Reminders

- Final project due June 3rd at 11:59 pm
- Submit
  - Presentation $\rightarrow$ Canvas dropbox
  - Expo article $\rightarrow$ Gradescope
  - Outreach $\rightarrow$ email; if it's a physical object, send me photos of it
- Please fill out the Course Evaluations before 6/8/23!
  - The math department uses this information to evaluate personnel and modify courses!
  - These evaluations are very important to my career as well as the TA's, as younger mathematicians.

# Presentation schedule: F 6/2 and M 6/5

| Date | Mathematician | Presentation Topic |
|---|---|---|
| F 6/2 | Camille Zeigler | TQFTs |
| M 6/5 | Run Wang | Low-complexity algorithms in knot theory |
| M 6/5 | Sunny Zhou | Knot theory and machine learning |
| M 6/5 | Noppakan Sirikul | Gesture theory and conducting |
| M 6/5 | Lawrence Cheung | Knots and financial market stability |
| M 6/5 | Catherine Nguyen | Physical knots |
| M 6/5 | Alisa Gao | Knot invariants |
| M 6/5 | Eli Lenett | Alexander horned sphere |
| M 6/5 | Diego Martinez | Hyperbolic knots |

# Presentation schedule: W 6/7

| Date | Mathematician | Presentation Topic |
|---|---|---|
| W 6/7 | Alice Li | Conway notation |
| W 6/7 | Yiwen Huang | Dowker notation |
| W 6/7 | Maximilien Gilli | Grid diagrams |
| W 6/7 | Vincent Hsu | Floer homology |
| W 6/7 | Wanzhu Zheng | Seifert surfaces |
| W 6/7 | Sam Kim | Knotted spheres |
| W 6/7 | Brian Kingsley | Knotted spheres |
| W 6/7 | Phyllis Law | Knotting in 5 dimensions |

# Presentation schedule: F 6/9

| Date | Mathematician | Presentation Topic |
|---|---|---|
| F 6/9 | Melody Guo | Unknotting number bounds |
| F 6/9 | Naomi Ladenburg | Torus knots |
| F 6/9 | Anthony Ramirez | Braids |
| F 6/9 | Genesis Navarro | Braids |
| F 6/9 | Chris Soriano De Jesus | Knots and DNA |
| F 6/9 | Mohammed Dharhan | DNA recombination |
| F 6/9 | Rishika Roy | DNA and enzymes |
| F 6/9 | Kledi Bici | Knot theory and DNA |
| F 6/9 | Ignacio Salazar | Knots and DNA |

# A biased tour of applications of knot theory

Melissa Zhang

UC Davis

Lecture 26

# Some topics covered this quarter

- knot: $S^1 \hookrightarrow \mathbb{R}^3$ (or $S^3 = \mathbb{R}^3 \cup \infty$)
- knot diagrams, Reidemeister moves, writhe of diagrams, positive/negative crossings, connected sum
- surfaces, 3-genus and 4-genus, knot concordance group
- linking number, Seifert matrices
- knot polynomials: Jones, Kauffman bracket, Alexander, HOMFLY-PT
- Khovanov homology (TQFTS, homology-type invariants)
- grid diagrams, the knot group

# There's much more to explore!

**Important thing 1**

This list, as well as the entire course, is from my point of view. We're all exploring a vast world with **little flashlights**!

knot theory $\subset$ low-dimensional topology, geometric topology

# Contact and Symplectic Topology

classical (Newtonian) mechanics

$\wr$

phase spaces, Lagrangian mechanics

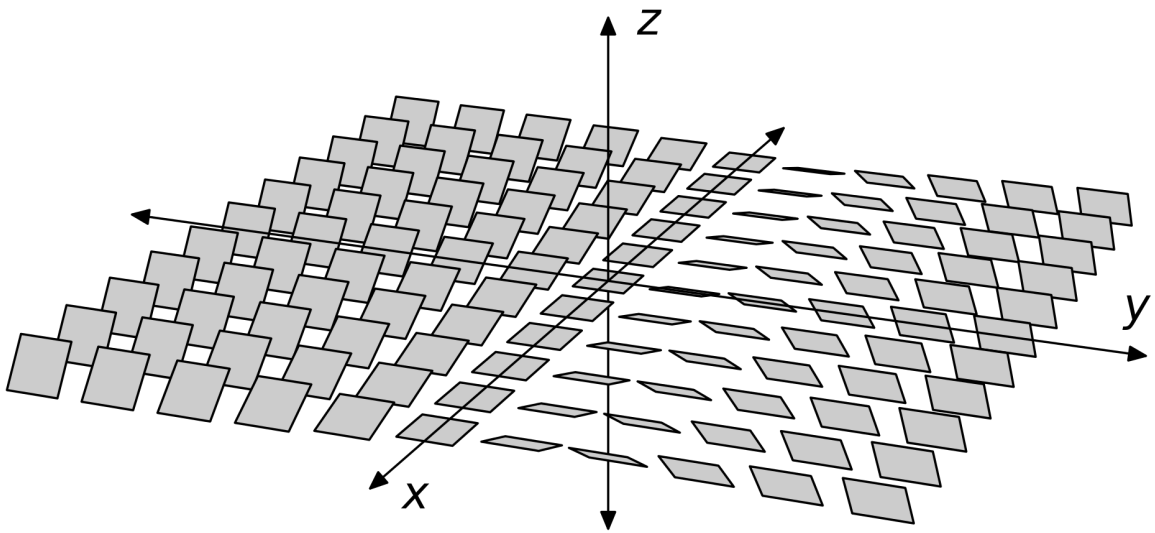
$\wr$

symplectic manifolds, Hamiltonian mechanics

- phase = (position, momentum) = $(\vec{x}, \vec{p})$
- $\Longrightarrow$ even-dimensional phase space
- **symplectic manifold** = 2n-dimensional manifold with a *non-degenerate 2-form*

# Contact and Symplectic Topology

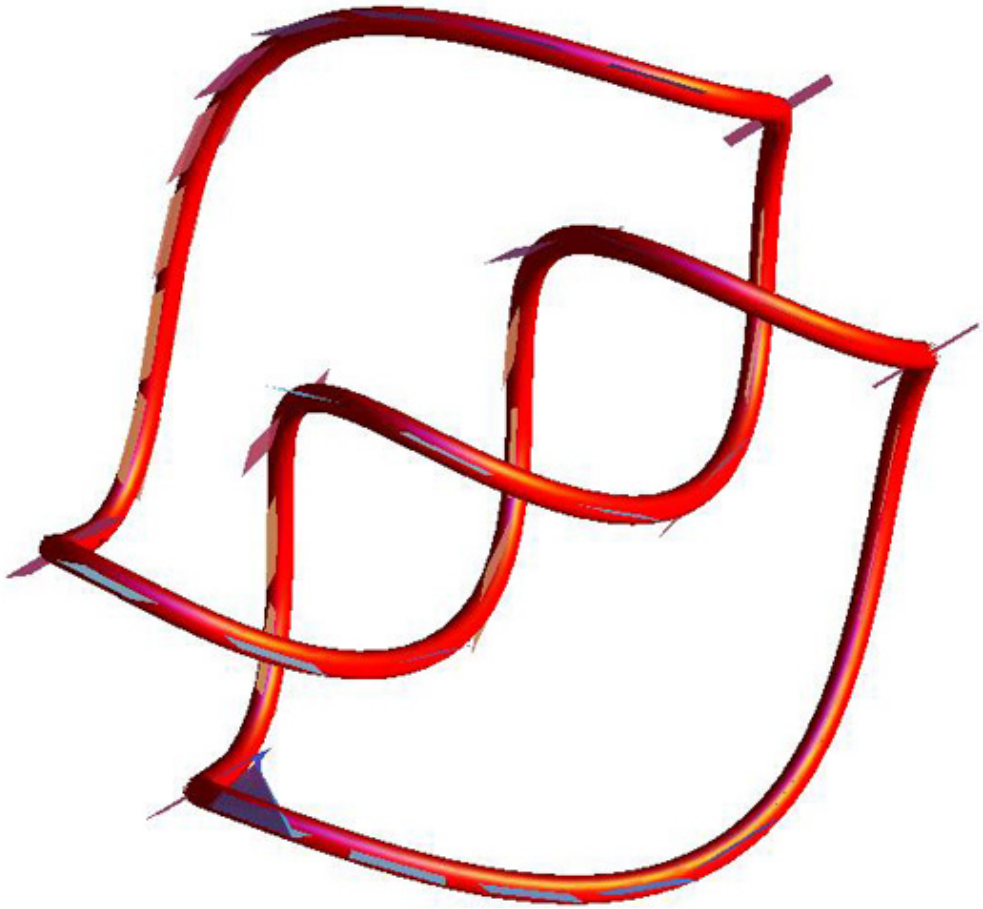
Odd-dimensional version = contact manifolds




https://en.wikipedia.org/wiki/Contact_geometry

# Contact and Symplectic Topology

A *Legendrian* knot in the *standard contact structure on* $S^3$:



`https://services.math.duke.edu/~ng/knotgallery.html`

# Contact and Symplectic Topology

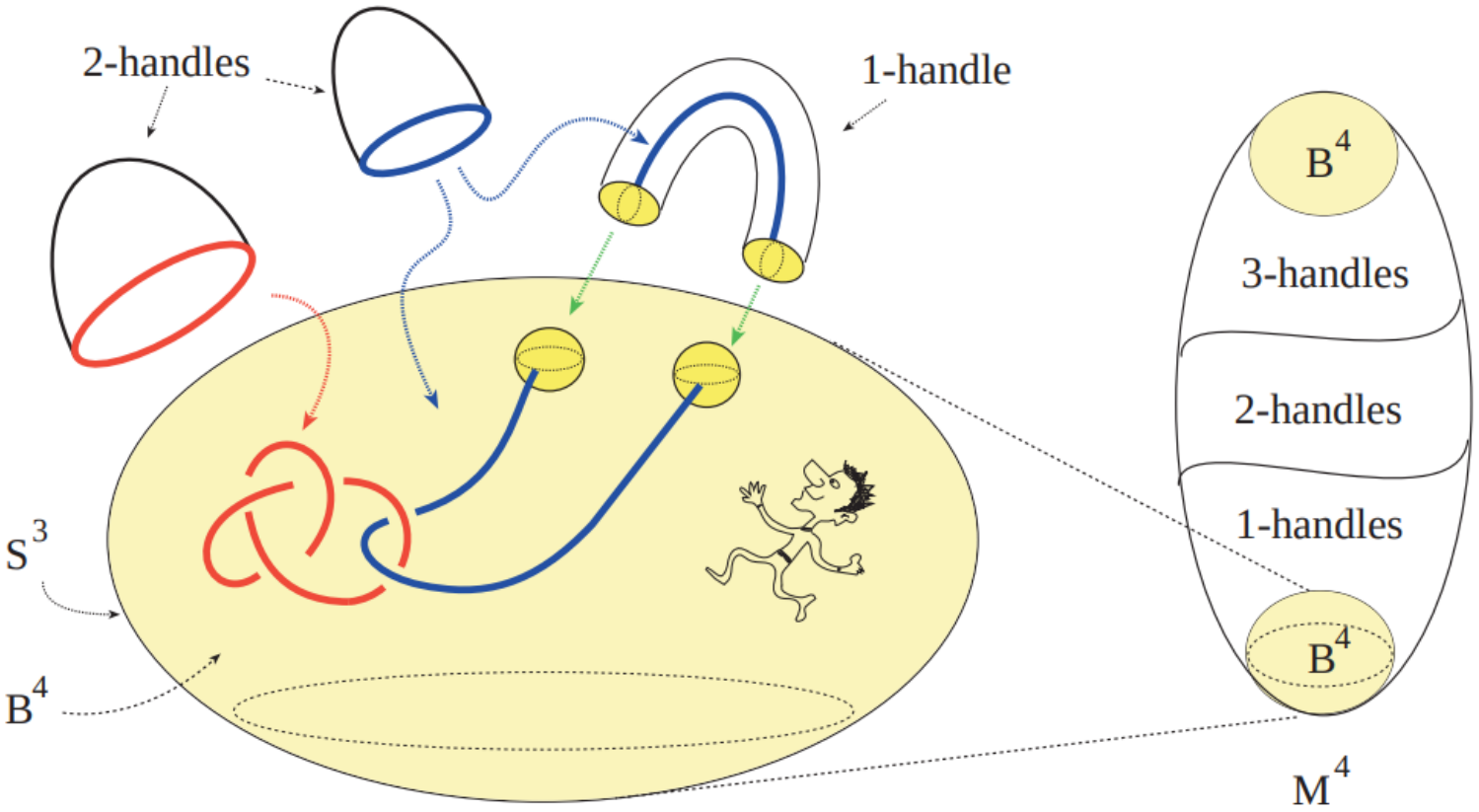
## Example

The *unit tangent bundle* of a manifold is a *contact manifold*.

- Car position $= \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$
- Car direction only (not speed) $= \theta \in S^1$

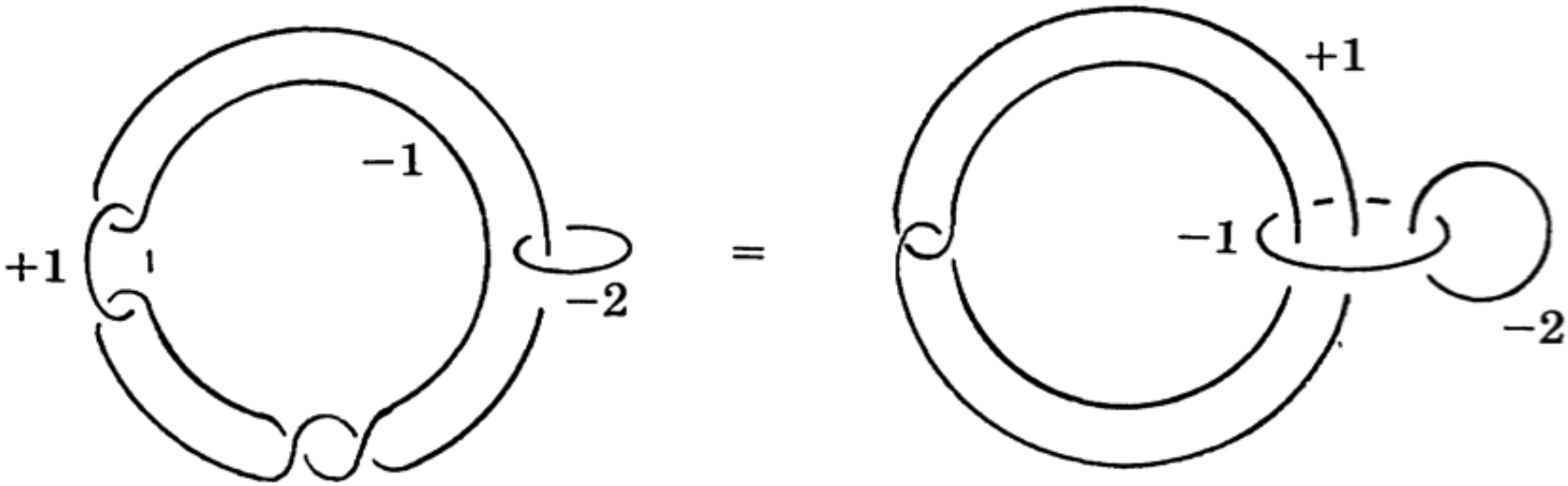
`https://services.math.duke.edu/~ng/knotgallery.html`

# Building 3- and 4-manifolds



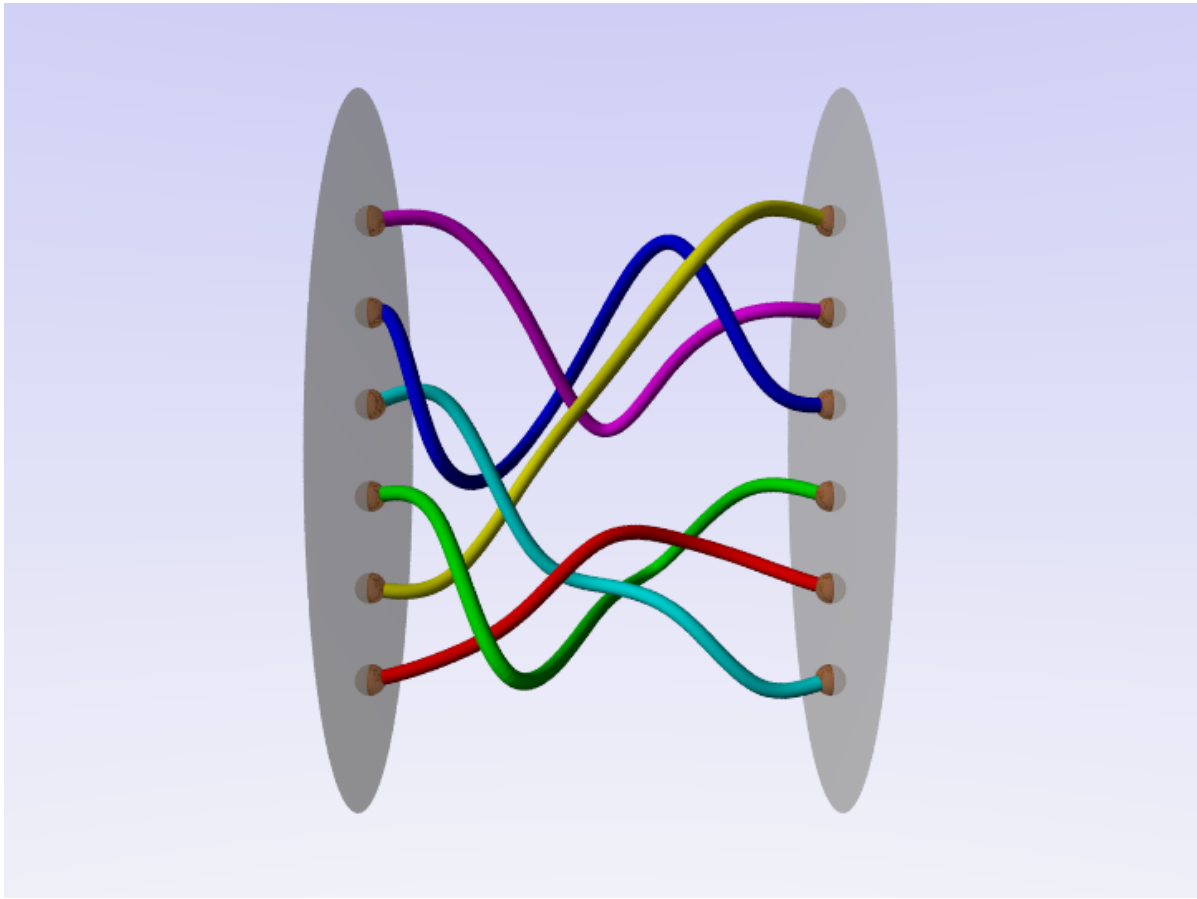

https://www-users.cse.umn.edu/~akhmedov/akbulut.lec.pdf

# Building 3- and 4-manifolds




http://www.selmanakbulut.com/papers/12.pdf

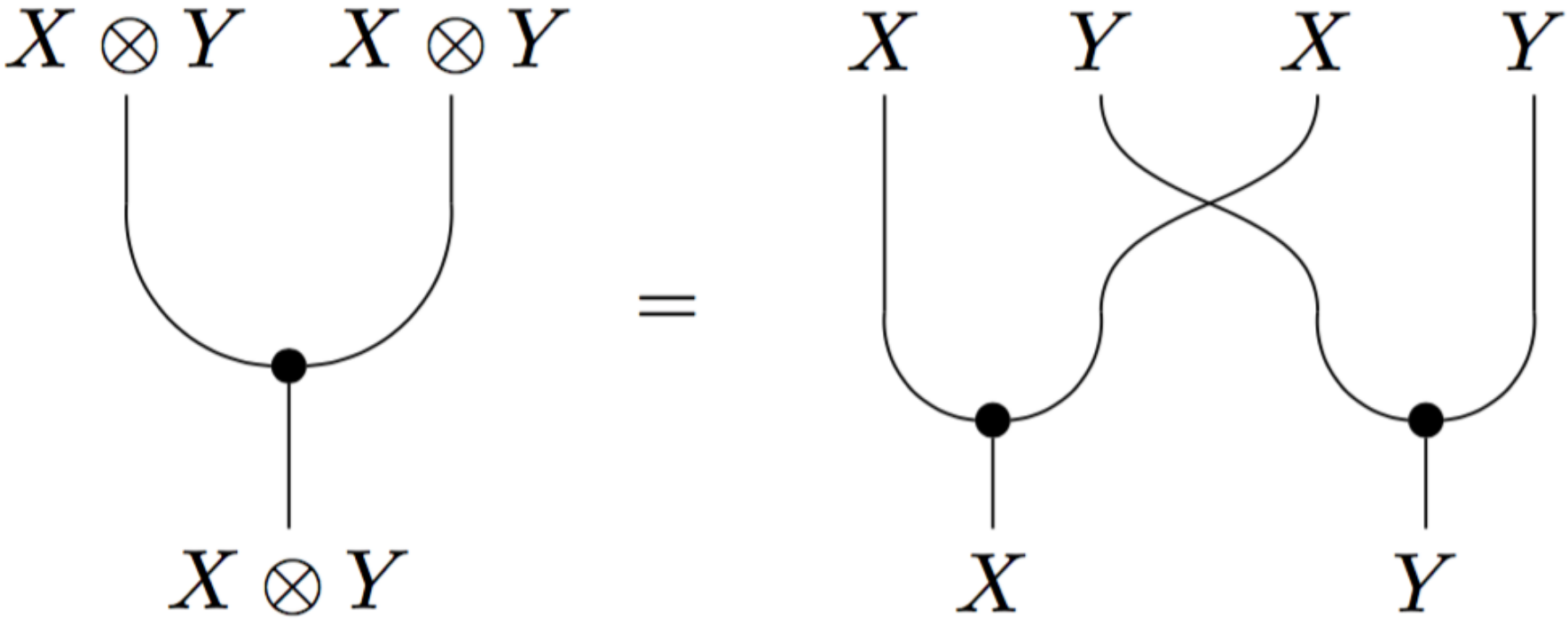
# Mapping class groups and dynamics



http://matematita.science.unitn.it/braids/summary.html

# Braided monoidal categories

The representation theory of various *Lie algebras* are packaged as *braided monoidal categories*.

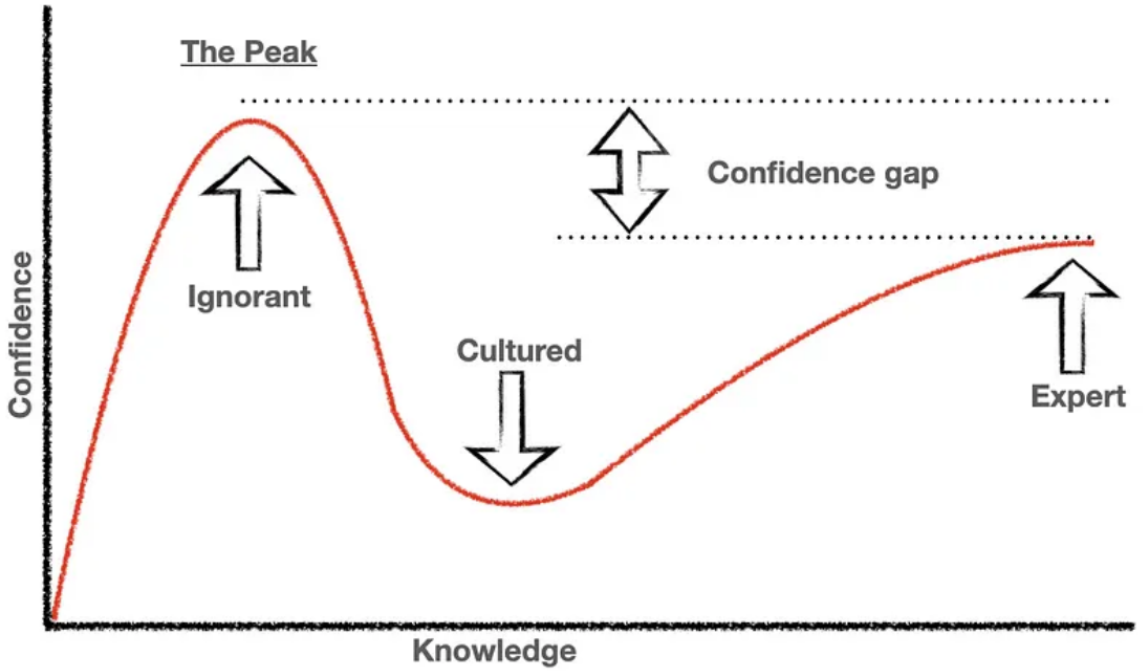



`https://math.stackexchange.com/questions/3590160/`

# More unsolicited advice

## Important thing 2

Just because you have heard the term for a concept does **not** mean you understand it. **Work out hard examples and really understand them.**




https://medium.com/geekculture/dunning-kruger-effect-and-journey-of-a-software-engineer-a35f2ff18f1a

# More unsolicited advice

But all that doesn't matter!

- Figure out and **clarify for yourself** what your goals are. (This takes time, and will also change with time.)
- Make sure your actions align with your goals. (This is hard.)
- **Have fun.** (This may not always be possible, but ideal will guide you.)

Best advice I got in grad school:

**Important thing 3**

**Do things out of excitement, not fear.**

– one of my PhD advisors, Eli Grigsby.[1]

[1] Definitely paraphrased; I never remember quotes exactly.