MAT180 HW02

(ADD NAME)

Due 4/14/23 at 11:59 pm on Gradescope

Reminder Your homework submission must be typed up in full sentences, with proper mathematical formatting. The following resources may be useful as you learn to use TeX and Overleaf:

- Overleaf's introduction to LaTeX:
  `https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes`
- Detexify:
  `https://detexify.kirelabs.org/classify.html`

How will this be graded? You will be graded for accuracy, but also effort. As such, you should attempt every problem, and even if you don't have the full solution, you should write down your thoughts (coherently, please). You are highly encouraged to collaborate with classmates! You must submit your own solution in your own words, though.

If you get stuck for a long time on a problem, talk to people! Discuss with classmates, go to office hours, etc. While it's an important skill to be able to keep at a problem, your time is also valuable; you know what's a reasonable amount of time for yourself to spend on a problem.

Exercise 1

(Ex. 1.2 in the book) Recall that the *crossing number* of a knot K is the minimal number of crossings needed in a diagram representing K. If the crossing number of K is c, then we say K is a *c-crossing knot*.

Prove that there are no two-crossing nontrivial knots.

SOLUTION.

Exercise 2

This is a multi-part exercise about the writhe of diagrams.

(a) Compute the writhe of the knot diagram shown in Figure 1.

(b) Describe how the writhe of a diagram changes under all the Reidemeister moves. (Note that there are two different Reidemeister 1 and 3 moves.)

SOLUTION.

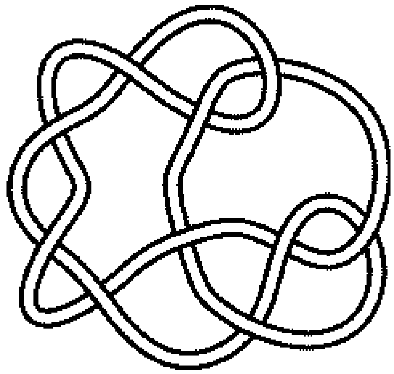

Figure 1: A diagram of the knot 10_{36}, taken from Knot Atlas.

Exercise 3

(1.17 in the book) Compute the absolute values of the linking numbers of the two links shown in Figure 1.39 of the book in order to show that they must be distinct links.

Solution.

Exercise 4

(1.25, 1.27 in the book)

(a) Show that the composition of any knot with a tricolorable knot yields a new tricolorable knot.

(b) Prove that the figure-eight knot (4_1 in the Rolfsen knot table) is *not* tricolorable. Conclude that the figure-eight knot and the trefoil knot are distinct knots.

Solution.

Exercise 5

(1.6, 1.7 in the book)

(a) Show that by changing the crossings from over to under or vice versa, any projection of a knot can be made into the projection of an alternating knot. (This isn't as easy as it might seem. How do you know your procedure will always work?)

(b) In a projection with n crossings, what is the maximum number of crossings that would have to be changed in order to make the knot alternating?

(c) Show that by changing some of the crossings from over to under or vice versa, any projection of a knot can be made into a projection of the unknot.

Solution.