MAT180 HW03

(ADD NAME)

Due ~~4/21/23~~ **Monday, 4/24/23** at 11:59 pm on Gradescope

Reminder Your homework submission must be typed up in full sentences, with proper mathematical formatting. The following resources may be useful as you learn to use TeX and Overleaf:

- Overleaf's introduction to LaTeX:
  https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes
- Detexify:
  https://detexify.kirelabs.org/classify.html

How will this be graded? You will be graded for accuracy, but also effort. As such, you should attempt every problem, and even if you don't have the full solution, you should write down your thoughts (coherently, please). You are highly encouraged to collaborate with classmates! You must submit your own solution in your own words, though.

If you get stuck for a long time on a problem, talk to people! Discuss with classmates, go to office hours, etc. While it's an important skill to be able to keep at a problem, your time is also valuable; you know what's a reasonable amount of time for yourself to spend on a problem.

Covered material This homework covers material from §2.4 and parts of §6.1-4 in the book. We will revisit some of the material in §6.1-4 again later in the course, in different contexts.

Exercise 1

In class, we sketched a proof showing that every knot diagram has a checkerboard coloring. Write down a proof of this statement in your own words.

SOLUTION.

Exercise 2

Show that if a diagram is alternating, then the associated signed planar graph has only one type of sign (either all $+$ or all $-$).

SOLUTION.

Exercise 3

Using the appropriate skein relations, compute the following polynomials for the figure-eight knot 4_1.

(It might be wise to include a figure for your resolving tree and only type up the part where you report the final polynomial you arrive at, or any polynomial calculations you do to arrive there. You do NOT need to type up the whole resolving tree!)

(a) Jones polynomial $V_K(q)$

(b) Alexander polynomial $\Delta_K(t)$

(c) HOMFLYPT polynomial $P_K(\alpha, z)$

(d) Then, verify that the HOMFLYPT polynomial specializes to the Jones and Alexander polynomials you computed.

Solution.

Exercise 4

Prove that for an amphicheiral knot K, the Jones polynomial is palindromic, i.e. $V_K(t) = V_K(t^{-1})$.

Solution.