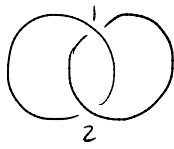


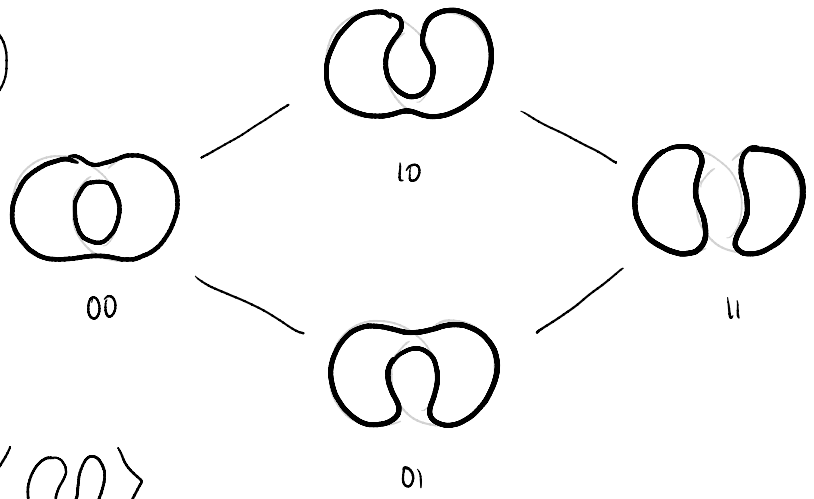
HW04



① $\langle \emptyset \rangle = 1$

② $\langle \times \rangle = \langle \rangle \langle \rangle - q \langle \text{X} \rangle$

③ $\langle L \cup O \rangle = (q + q^{-1}) \langle L \rangle$



$$\langle \emptyset \rangle = \langle \text{O} \rangle - q \langle \emptyset \rangle$$

$$\langle \text{O} \rangle = \langle \text{O} \rangle - q \langle \text{O} \rangle = (q + q^{-1})^2 - q(q + q^{-1})$$

$$\langle \emptyset \rangle = \langle \emptyset \rangle - q \langle \text{O} \rangle = (q + q^{-1}) - q(q + q^{-1})^2$$

$$\begin{aligned} \Rightarrow \langle \emptyset \rangle &= \left((q + q^{-1})^2 - q(q + q^{-1}) \right) - q \left((q + q^{-1}) - q(q + q^{-1})^2 \right) \\ &= (q^2 + 2 + q^{-2} - q^2 - 1) - q(q + q^{-1} - q(q^2 + 2 + q^{-2})) \\ &= (1 + q^{-2}) - q(q + q^{-1} - q^3 - 2q - q^{-1}) \\ &= (1 + q^{-2}) - q(-q - q^3) \\ &= 1 + q^{-2} + q^2 + q^4 \\ &= q^4 + q^2 + 1 + q^{-2} \end{aligned}$$

$$\begin{aligned} \ln H^t, n_+ = 2, n_- = 0. \Rightarrow \tilde{V}_{H^t}(q) &= (-1)^{n_-} q^{n_+ - 2n_-} \langle \emptyset \rangle \\ &= q^2 \langle \emptyset \rangle = q^6 + q^4 + q^2 + 1. \end{aligned}$$

$$\begin{array}{r} q^5 + q \\ q + q^{-1} \overline{) q^6 + q^4 + q^2 + 1} \\ \underline{-(q^6 + q^4)} \\ 0 \\ \underline{-(q^2 + 1)} \\ 0 \end{array}$$

$$\Rightarrow V_{H^t}(q) = q^5 + q. \quad (\text{Indeed, there is a sign convention issue!})$$