MAT180 HW05

(ADD NAME)

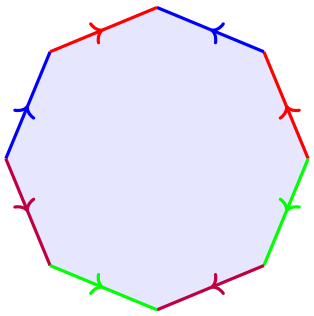
Due 5/5/23 at 11:59 pm on Gradescope

Reminder Your homework submission must be typed up in full sentences, with proper mathematical formatting. The following resources may be useful as you learn to use TeX and Overleaf:

- Overleaf's introduction to LaTeX:
  `https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_minutes`
- Detexify:
  `https://detexify.kirelabs.org/classify.html`

Exercise 1

We mentioned in class that a genus-2 orientable surface could be constructed by gluing the edges of an octagon together:



Sketch a series of pictures showing how to see that, after gluing up all the same colored edges according to the orientations shown, you will have the standard picture of a genus 2 surface.

Exercise 2

In this exercise, you will explore the 3-genera of 7-crossing knots.

(a) Using Seifert's algorithm, draw Seifert surfaces for all seven 7-crossing knots in the Rolfsen knot table (available in the back of your book, or at `http://katlas.org/wiki/The_Rolfsen_Knot_Table`).

(b) Now compute the genera of the surfaces you drew.

(c) Compare the genera of the Seifert surfaces you drew with the known 3-genera of the 7-crossing knots, which you can access by using KnotInfo: `https://knotinfo.math.indiana.edu/index.php?isdesktop=1`. (At the top of the page, select 7 crossing knots; then find the "`Genus-3D`" property and check the box. Click `Submit` to see the results.)

Exercise 3

In Section 4.3 of the book, around page 100, Adams proves:

Theorem. The 3-genus of knots is additive, i.e. $g_3(K\#J) = g_3(K) + g_3(J)$.

Read the proof, and then provide an *executive summary* of the proof.

Executive summaries In mathematical writing, proofs must be extremely rigorous, and hence quite long. In research articles, we often first give a shorter summary of the proof of our main results, so that an expert reader can get the main gist of the steps of the proof, without getting bogged down by the details. Your summary should

- be shorter than the actual proof,
- describe all the main steps and insights from the actual proof, including how to resolve any difficulties that arise,
- and convince someone familiar with the field that they could reconstruct the whole proof if given enough time.